CERME 8 Antalya, 6-10 February 2013

8th congress of European Research in Mathematics Education

WG1 report

ARGUMENTATION AND PROOF

Sunday, February 10th 2013

19 papers : France (5) – Germany (5)) - USA (2) – Canada (2) – Italy (1) - Greece (1) –Norway (1) - Algeria (1) – Tunisia (1

1 poster (France)

27 participants : Germany (6) France (6) Italy (4) Greece (2) Norway (1) UK (1) Algeria (1) Tunisia (1) USA (3) Canada (2)

Menu of each session

- A main question that will be discussed - Each presenter is requested to focus on the main question
- 10 minutes for each presentation, followed by 5 minutes for precise questions for clarification if needed.
- A collective discussion

Presenters are invited to post their presentation in the WG1 dropbox.

Session 1 – Wednesday February 6th - 17:30 – 19:00

Epistemological and didactical issues and their relationships in proof and proving in mathematics education

Chair : Samuele Antonini

Renaud Chorlay (France)
The making of a proof-chain: epistemological and didactical perspectives

Judith Njomgang Ngansop and Viviane Durand-Guerrier
(Cameroon and France)
0, 999… = 1. *An equality that questions the relationship between truth and validity*

Collective discussion - Gila Hanna

According to Lakatos, in the construction of a mathematical theory, proving and defining are intertwined processes. Relying on your research and experience, what are the epistemological and cognitive aspects involved in the relations between proving, defining, truth and validity?

Proving and defining

Mathematical theories: products and processes

Different aspects of a notion can be used to define

Important interplay between defining and proving

- Within a deductive chain, definitions come first, although they usually developed last **(**Proving may lead to discover/ identify unexpected properties of mathematical objects)

Proving and conceptualization

Have students access to the key ideas of mathematical theories?

Should proofs be connected with intuitive arguments? (If theoretical arguments do not have a counterpart in intuition, it is not surprising that products are counter-intuitive)

How can argumentation promote conceptualization?

Truth and validity: a logical and epistemological view

Truth depends on interpretations (i.e. Non-Euclidean geometry)
("The sum of angles in a triangle is 180°": the truth depends on which underlying theory is chosen)

Validity: proof in a system of axioms
- at least a local theory of axioms
- necessity of definitions

Problem: get the statement to be theoretically true also in the theory when one believes it to be true. The theory must suit the interpretation.

Truth and validity: a cognitive view

Different use of the terms true/false, valid/invalid in classroom (for proof, calculations, etc.)

What is the perception of validity by the students? Is it connected to proof?

Truth: difference between something linked to physical objects and abstract objects.

Some students don't accept the truth even after accepting the proof.
Some students don't matter whether they believe it or not, it is proved
Some students are convinced by faulty proofs.

Important: giving opportunity to students to discuss about validity, truth, defining and proving in a theory.

Session 2: Thursday February 7th - 8:30 – 10:30

The role of logic and language in teaching, learning and analysing proof and proving process - Issues at secondary level

Chair: Viviane Durand-Guerrier

Jenny Cramer (Germany)
Possible language barriers in processes of mathematical reasoning

Zoe Mesnil (France)
The role of logic in analysing mathematical discourse: a proposal for training for teachers

Christavgi Triantafillou (Greece)
The nature of argumentation in school texts in different contexts

Collective discussion – Francesca Morselli

Language and Logic Connections

- Language and logic (deductive reasoning) are closely related.
- Language can be barrier in a multi-lingual learning environment.
- Logic is a theory for modeling human reasoning.
- An aim for logic: to control the validity of proof?
    - *Possible reason to teach logic*.

Language and Logic Connections

- Respective role of logic and language in conceptualization of mathematics.
- There exist positive and negative effects of logical analysis in multicultural context.
  - It can exclude or discriminate students.
  - It can allow a bridge between mathematical language.

Logic or not logic ?

- There is a balance between the necessity of mathematical language, and the importance of remaining close to natural language.
  - We are far from a consensus....*to be continued.*
- Logical competence versus logic as a body of knowledge.
- Abstraction and logic may not be so strictly related.

What logic is needed for teachers?

- Teachers need to be exposed to logic, however, the question remains how it ought to be done.
- Logical proof may be considered in terms of a final product, as well as a process in action.

Poster session: Thursday February 7th 14:00 – 15:30

Foyer – Ground Floor

Simon Modeste (France)

Modelling Algorithmic thinking: the fundamental notion of problem

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During the poster session, Denise Grenier invites the participants to experiment a mathematical SiRC (*Foyer – Ground Floor*)

Session 3: Thursday February 7th – 16:00 – 18:00

The role of logic and language in teaching, learning and analysing proof and proving process - Issues for advanced mathematical thinking

Chair : Samuele Antonini

Nadia Azrou (Algeria)

Proof in Algebra at the university level: Analysis of students' difficulties

Faiza Chellougui and Rahim Kouki (Tunisia)

Use of formalism in mathematical activity - case study: the concept of continuity in higher education

Eva Mueller Hill (Germany)

The epistemic status of formalized proof and formalizability as a meta-discursive rule

Collective discussion – Annie Selden

According to Quine, formalisation in predicate calculus contributes to conceptual clarification. However, it appears that for many students, formalization is an insuperable obstacle. Are there any aspects of your own research that bring light on this contrastive landscape?

Logic and formalism

- Formalization is essential for mathematical work (formalization to control the correctness)
- Formalization "is a tool, not a game"
- Different aspects of formalization
- - logic formalization (quantifier)
- - mathematical formalization (i.e. algebra)
- - formalization is also the choice of aspects of a concept that fit in a theoretical way (example: Perpendicularity, continuity). Two equivalent definitions can lead to different cognitive processes and difficulties

Logic and formalism

- In mathematics education: understanding formalization as a process is essential

Some educational aspects:

- relations between different formalizations and students' difficulties.
- different formalizations in different textbooks.

Syntax and semantics

- There is a semantic context to syntactic work. (how does a machine know what to do next when presenting a proof?)
- Can students understand what a mathematical object is (i.e. group) without examples?
- The clue point is the dialectic between syntax and semantic
- When the semantic doesn't control the syntax, it evolves into a new semantics (of the syntax) which will guide the reasoning forward.

Session 4: Friday February 8th – 9:00 - 10:30

Designing activity fostering argumentation and proof skills - Research situations at various level

Chair: Viviane Durand-Guerrier

Patrick Gibel (France)
The presentation and setting up of a model of analysis for levels of proof in mathematics lessons in primary schools

Denise Grenier (France)
Research Situations to learn logic and various types of mathematical reasonings and proofs

Collective discussion – Reinert Rinvold

In which respect the models you have elaborated are sound supports for developing new research situations oriented to the development of proof and proving skill?

What is the use of models ?

For researcher , the models are useful:

- For developing situation :
- For predictng and describing the students activity
- For analysing student’s reasoning, theri evolution
- for communicating with teachers and eductaors
- For trying to promote students’ engagement in proving

Dialectics Action – formulation - Validation

How can we identifiy relevant mathematical problems (e.g. optimisation – number theory)

The importance of the organisation of the situation (dialectics)

The function of reasoning :

- To take decision
- To open possibilities
- To express and discuss methods

Does one can learn *concept* and *how to prove* at the same time ?

Difference between mathematicians and students

A matter of priority : to focus on proof and proving skills. How to evaluate these skills ?

You should acquire the concept before – learning with meaning.

Proof is constitutive of a concept; It is partly content dependant.

Both aspects are important.

The teacher's role

- Favouring the engagement of students in the problem
- Giving the possibility to students modify the problem
- Organising the students' exchanges
- Introducing relevant elements (new question – new game – variantes etc..) to allow the evolution of the methods and to favour the production of proof, including counter examples.

How to evaluate proof and proving skills

- A question : do students recognize it as a proof when they work together ?
- It may occur that for teachers something is a proof, but not for students, and the opposite as well
- Along a dedicate course to research problems, evaluating on such problems.

Session 5: Friday February 8th – 11:00-12:30

Designing activity fostering argumentation and proof skills - Teachers' development issues

Samuele Antonini

Leander Kempen and Rolf Biehler (Germany)
Students' use of variables and examples in their transition from generic proof to formal proof

Margo Kondratieva (Canada)
Multiple proofs and in-service teachers' training

Ruthmae Sears (USA)
A case study of the enactment of proof tasks in high school geometry

Collective discussion – David Reid

Based on your own research, what should in-service teacher and pre-service teachers experience in order to be able to face the educational challenge of allowing students to develop proof and proving skills?

Teachers

Very negative beliefs and attitude towards proof from students who want to become math teachers.

What we can do to offer support to teachers to modify their behavior? To learn from experience?

Knowledge and experience

Gap between knowledge and experience: teachers do not necessarily take into practice what they experienced and discussed at the university level.

Big problem: how to make the teacher become a cultural agent in school (self-reflection is important but something more is necessary)

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Tools for observing

Tool for observing: example: Recognizing generic proof is really important for teachers (use of models for observing)

The question how you know that something is generic, what is it about this that makes it generic, is complicated. Different people see different things in examples.

General and particular

- With generic proof, we have two kinds of problems. Students have problems in:
- 1) seeing the generality in the specific examples,
- 2) communicating this generality. (Even if students are writing the correct algebra we cannot be sure that they are aware of the generality)

Session 6: Saturday February 9th 8:30 – 10:00

Theoretical perspectives on reasoning, proof and proving - New theoretical perspectives on proof and proving (1)

Chair: Viviane Durand-Guerrier

Gila Hanna (Canada)
The width of proof

Francesca Morselli (Italy)
Approaching algebraic proof at lower secondary school level: developing and testing an analytical toolkit.

Annie Selden & John Selden (USA)
Persistence and self-efficacy in proof construction

Collective discussion – Patrick Gibel

What are the more prominent aspects of your theoretical approach that should interest the research community involved in proof and proving in an educational perspective?

What can we learn for matheducation from (brillant) mathemacians ?

- Importance of conceptual (non formal) proof
- Communication and intrinsic properties of a proof are different things.
- Importance of what is memorized (concept and methods) – of mathematical knowledge
- Mathematicians draw on a repertoire of proof when they proof. To get a repertoire, you need to collect different proofs.

What can we learn for matheducation from (brillant) mathemacians ?

- Persistence and self-efficacy in proof construction
- Potential useful actions for the problem-solving part: Exploring – Reworking – Validating (A proof control at a meta level) .
- The importance of the long term research for solving a problem.

Crossing a comprehensive theoretical framewok with the cycle of Algebra

- Habermas rationality.
- Algebraïc transformation as a tool to prove.
- Numerical problems that can be solved by elemenatry Algebra (the sum of three consecutive numbers)
- Necessity for students of a goal-oriented approach for algebraïc transformation.

Session 7: Saturday February 9th -16:30 – 18:00

Theoretical perspectives on reasoning, proof and proving - New theoretical perspectives on proof and proving (2)

Chair: Samuele Antonini

David Reid (Germany)
Biological bases for deductive reasoning

Reinert Rinvold and Andreas Lorange (Norway)
Multimodal proof in Arithmetic

Markus Ruppert (Germany)
Ways of analysing reasoning-thought processes in an example based learning environment

Collective discussion – Leander Kempen

What are the more prominent aspects of your theoretical approach that should interest the research community involved in proof and proving in an educational perspective?

Deduction and analogy

Proving is done by human beings. Understanding the origins of our ability to prove might help us to identify important research questions related to proof and proving.

What is the basic nature of human thinking?

When and why do people think deductively? How people use analogical reasoning?

Deduction and causality.

Connection between language and deductive reasoning (access to language leads to a higher level of abstraction)

Deduction is inherent to our cultural construction.

Multimodality

Perception in arithmetic

Multimodal proof (more than one modality used simultaneously: ex. in arithmetic).

Multimodality in arithmetic: proof and explanation
visualization of generality

To continue ….

10/02/13