MODELLING ALGORITHMIC THINKING : THE FUNDAMENTAL NOTION OF PROBLEM

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The notion of a problem plays a very important role in algorithmic thinking. We propose a definition of 'problem' which is suitable to model this thinking. We will show how it allows us to express the tool-object duality and describe conceptions.

In many countries, algorithms and algorithmics are taking a growing importance in the curricula of mathematics[1]. This appearance of algorithmics raises many questions (Modeste & Ouvrier-Buffet, 2011). Indeed, algorithms are mathematical concepts shared with computer science. Their present-day role in mathematics and their place in the mathematical activity in the classroom have to be questioned. It also appears that the concept of algorithm is strongly linked with the mathematical proof.

So, it seems important to have a better understanding of advanced algorithmic thinking and how it interplays with advanced mathematical thinking (Tall, 1991, Harel & Sowder, 2005). We propose a model of advanced algorithmic thinking from an epistemological point of view, based in particular on Knuth (1996) and Chabert (1999). This model permits to study how algorithmics is transposed in different institutions (curricula of mathematics, curricula of computer science, textbooks...). One important point in this model is the notion of problem.

WHAT IS A PROBLEM?

We propose to adapt a definition which comes from the theory of algorithmic complexity. Giroud (2011) also used such a definition to describe the concept-problem in problem solving activities. Since this definition, a **problem** (e.g. finding the gcd) is:

- ***I*** a set of **instances** (e.g. $\mathbb{N}^2$, all the pairs of two integers)
- ***Q*** a **question** about these instances (e.g. what is the gcd of the 2 integers?)

This definition is perfectly suitable for algorithmics: an algorithm is a systematic method which must give answer to a question, *for all instances of the problem*, and after a finite number of steps (e.g. Euclid's algorithm solves the problem of gcd for any 2 integers). A problem is *instantiated* when one choose a particular instance ***i*** and try to answer the question ***Q(i)*** for this particular case (e.g. what is the gcd of 3654 and 76?).

WHAT FOR?

First, this definition allows us to rephrase the tool-object duality (Douady, 1986): Algorithm is a tool when used to solve a problem. Algorithm is an object when algorithms are in the instances or when questions involve algorithm.

It also permits to use the cK¢ model (Balacheff, 1995) to describe conceptions in the academic knowledge (μ-conceptions) for algorithm in order to model algorithmic thinking and also to analyse people's or institutions' conceptions.

Finally, it permits to characterise problems with potential for the learning of algorithmic thinking, in order to design and study teaching situations.

CONTENTS

In the poster, we will describe with more details the relation between this notion of problem and algorithmic thinking. We will introduce the model of conceptions and show how it fits with the notion of problem and the tool-object duality.

Finally, we will show how the model can be used to study the didactical transposition, based on the case of French curricula and textbooks for high school, and particularly on the way *dichotomy* is treated.

NOTES

1. The National Council of the Teachers of Mathematics dedicated his 1998 Yearbook to this issue (NCTM, 1998). since 2003, algorithmics has been introduced in high-school curricula of mathematics in France.

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