POSSIBLE LANGUAGE BARRIERS IN PROCESSES OF MATHEMATICAL REASONING

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The fostering of deductive reasoning within mathematical argumentation processes is a demanding task for teachers. Deductive reasoning requires not only awareness of the epistemic value of statements but also of the structural components of mathematical knowledge. The relations between different statements become visible within mathematical arguments, and the truth of propositions is established independent from their current context. The aim of this paper is to show that this requires a certain language register, characterized by context-independency and precision, to which learners in school have different levels of access.

Mathematical reasoning, academic language, migration background

INTRODUCTION

The introduction of reasoning and proof into the NCTM standards 2000 triggered a focus on argumentation and proving in the curriculum in many countries. As mathematical reasoning is closely connected to the exploration of connections between new statements and existing mathematical knowledge, it seems to be a promising way to promote learning. More reasoning in school thus appears to be a welcome development. Research on problem solving (Lubienski 2000, 2004) and modelling (Leufer and Sertl 2010) however pointed out disadvantages for children from lower socioeconomic backgrounds and linked these findings to Bernstein's theory of different access to the language register required in education. Knipping (2012b) points out that so far, there has not been any research on the question whether similar effects can be observed for argumentation.

Limited access to the language register required in school contexts can be seen as a possible explanation of the PISA 2000 results in Germany, which showed huge deficiencies in the performances both of students from a lower socioeconomic background and of children with migration background. In this paper I will first explore the nature of deductive reasoning and its possibilities for the learning of mathematics. Secondly, I will focus on the language required in educational contexts and the possible obstacles it holds for students from a lower socioeconomic background and learners for whom the language of education is not the mother tongue. After that, I will establish a connection between the characteristics of mathematical reasoning and the features of academic language. Finally, I will present my research approach that aims at facilitating the integrated learning of mathematical reasoning and the academic language register.

ARGUMENTATION, PROOF AND DEDUCTIVE REASONING

Much has been said about the "complex, productive and unavoidable" (Boero, 1999) relationship between argumentation and proof. Following Toulmin (1958), proof can be considered as a special form of argument. Toulmin sees arguments as steps from a datum or a set of data to a conclusion, justified by a warrant for which backing may be produced if necessary. Mathematical proofs follow the same structure, ideally relying on axioms or established mathematical knowledge as data, and using the rules of logical deduction in order to arrive at a conclusion. In mathematical practice however, Hanna and de Villiers (2012, p.3) state that "a proof is often a series of ideas and insights rather than a sequence of formal steps". For academic mathematics, the scientific community has been negotiating the rules for proving for a long time, and mathematicians generally have an idea about which steps in a proof they may omit (Knipping 2012b).

For school mathematics however, Knipping (2012b, p.4) points out that "there are no previously negotiated criteria for argumentation". These criteria need to be agreed upon by the community in which argumentation takes place. School mathematics cannot take an axiomatic approach to proving, or assume that the students know which types of deduction are acceptable. The role of the social community is crucial for argumentation processes. I will follow Knipping's definition, which takes the social community into account and defines argumentation as "a sequence of utterances in which a claim is put forward and reasons are brought forth with the aim to rationally support this claim" (Knipping 2003, p.34, my translation). This definition encompasses proof and deductive reasoning.

As pointed out before, mathematical argumentation can be seen as a continuum, reaching from very informal arguments and inferences based solely on the authority of the speaker to strictly logical proof. These kinds of arguments differ significantly in the way in which they use deductive inferences. Not every argumentation in mathematics contains deductive reasoning. Aberdein (2012) sees mathematical argumentation as consisting of two parts. It is characterised by an underlying inferential structure, which follows strictly logical criteria, and an argumentation, which is the visible part of the argument. The argumentational part seeks to convince others that logical criteria have been obeyed and that the given inferences are valid. In order to account for this two-layered view on mathematical argumentation, Aberdein has suggested a categorization of arguments into different schemes. Arguments in which every step is a deductive transmission from the premises to a conclusion are characterized as A-scheme arguments, as the argumentational structure and the inferential structure are directly connected. Mathematical proof falls into this category, as justifications based on logical deduction are given for each step of the argument. If the connection to the inference structure is less explicit, but could theoretically be split up into a limited number of deductive steps, Aberdein classifies the argument as B-scheme. This type of argument is based on deductive reasoning; however, intermediate steps may be omitted or references to statements proven elsewhere may be included. In academic mathematics this is often the case in proofs presented in mathematics journals, where some steps are omitted and left for the qualified reader to complete. Deductive reasoning is a prerequisite for A- and B-scheme arguments as it ties the logic of the inferential structure to the visible argumentation. The last category for arguments proposed by Aberdein is the C-scheme. All arguments without direct or indirect reference to the inferential structure are contained in this category. Typical for this category are visual arguments and other informal argumentation techniques which make no use of deductive reasoning.

Mathematical reasoning and learning mathematics

As all of the presented sorts of arguments occur within mathematics as an academic discipline, school mathematics needs to face the question, which kinds of argumentation it wants the students to engage in. C-scheme arguments can be very helpful in conjecturing processes, as they lead to assumptions about possible hypotheses. In order to systematize new mathematical knowledge however, some connection to an inferential structure must be established. Arguments that are solely based on informal practice or intuition cannot show what the truth of statements and their relationships depend on. They do not explain why a statement is true. However, Hanna (2000), de Villiers (1990) and others have pointed out the importance of the explanatory potential offered by a proof based on deductive reasoning. The direct link to the inferential structure also simplifies the systematization of new knowledge into the existing internal mathematical knowledge structure.

In recent years, there have been many attempts to make proving more accessible to students. As deductive reasoning is necessary for proving, much about its fostering can be learned from these approaches. Boero, Garuti, Lemut and Mariotti (1996) introduced the concept of cognitive unity between conjecturing and proving. This concept states that proving becomes easier if during the conjecturing phase, students discover arguments that they can later use in the proof. Suitable tasks must be chosen in order to enable the discovery of important arguments. Knipping (2012b) compares the restructuring of arguments found in the conjecturing phase into a deductive chain to a transition from Aberdein's C-scheme to B- or A-scheme arguments.

Another approach for dealing with proofs in school was developed by Hanna and Jahnke (2002). In this approach, the importance of hypotheses for proving is emphasized, and Freudenthal's concept of local organisation plays a major role (Hanna and Jahnke 2002, p.3). At the beginning of a new exercise, the learners are engaged in measuring and experimentation, which leads to a speculation about possible hypotheses. When several hypotheses have been collected, the students are asked to create connections between them and put them into a structure, thereby establishing a local order. Jahnke (2009) emphasizes the importance of inferences for mathematics. With Aberdein's scheme, the established structure between the hypotheses based on the newly created local order can be characterized as B-scheme or A-scheme arguments. Emphasis is not put on the validity of the hypotheses but on the certainty of the inferences. The learners work out a deductive chain that is valid as long as the hypotheses are true.

The specification of starting conditions and the insight that statements are dependent on other statements are characteristics of mathematical reasoning, and especially of deductive reasoning. Reasoning which has its origins in an inferential structure requires awareness of the mathematical background of theorems. Having established the connection of a new statement to existing mathematical structures, further exploration becomes possible. Bikner-Ahsbahs et al. (2011) have pointed out the potential of mathematical reasoning for learning mathematics. Reid (2001) has described how deductive reasoning plays a role in the acceptance of explanations. Connections and links between new knowledge and existing knowledge are forged, and bridges between previously independent islands of knowledge are established by mathematical reasoning. Schoenfeld (1994, p.68) claims that "looking to perceive structure, seeing connections, capturing patterns symbolically, conjecturing and proving, and abstracting and generalizing" are fundamental to mathematics. All of the processes mentioned in the quote are also important in mathematical reasoning. Thus, promoting the ability to reason deductively seems to be a promising path in order to enable students to learn mathematics.

Possible obstacles in the teaching of mathematical reasoning

However, more deductive reasoning in schools may also trigger some unwelcome effects. Lubienski (2000) has pointed out that not all students benefitted equally from the greater emphasis that was put on problem solving in recent years. Her findings showed that while learning was fostered for both students from lower and students from higher socio-economic backgrounds, the open and context-embedded material increased the gap between the two groups. The group of students from a lower socioeconomic background was much slower in their progress and often felt insecure about the acceptability of their arguments. Many of the students uttered the wish for more support by the teacher. Furthermore, the arguments brought forth by these students were often directly linked to the context given in the task, without focussing on the intended mathematical background.

For mathematical modelling, similar problems were observed by Leufer and Sertl (2010). The application of mathematics on realistic problem situations was supposed to increase motivation and bridge the gap between school and real life, especially for students from lower social classes. However, especially these students had problems in solving the given tasks.

A possible explanation for these differences in achievement between students with higher and lower socioeconomic status given by Lubienski and by Leufer and Sertl is based on Bernstein's sociology of education. According to Bernstein (2003), the social class of the speaker influences the language register he or she is capable of, and likely to be, using. Bernstein distinguishes between restricted and elaborated codes. The different codes are characterized by specific discourse forms and different conversation modes. Language of a restricted code takes place in situations of temporal and spatial proximity. The discourse can be classified as horizontal (Bernstein 1999), is dependent on the immediate context in which it is spoken and can, while coherent in one given context, be illogical across different contexts. Elaborated code, on the other hand, is characterized by its reference to objects that are not necessarily tangible. It makes use of vertical discourse, which is marked by context-independency, coherence, and the ability to abstract from concrete objects. Bernstein (2003, p.109) describes that, while restricted code appears in all social classes, children from are working class background are often limited to this type of language. In contrast to this, children from the middle and higher classes experience the usage of both restricted and elaborated code at home.

The considerations of Bernstein explain general deficiencies of children from a lower socioeconomic background in school, as elaborated code usually is the required language register in the educational context. In order to see which kind of language is demanded in a certain situation, Knipping (2012a) describes Bernstein's approach of necessary recognition and realisation rules. The usage of real-world contexts in mathematical tasks impedes the recognition of the expected vertical discourse. These problems of children from a lower socioeconomic background have not been analysed with a focus on argumentation yet. Mathematical reasoning requires many processes of abstraction and generalization. It can be expected that children with limited access to vertical discourse encounter problems.

ELABORATED CODE AND SECOND LANGUAGE LEARNING

Bernstein's theory of linguistic codes is concerned with children from different socioeconomic backgrounds. Social class, however, is not the only factor for language learning and achievement in school.

The results of PISA 2000 forced the German educational system into becoming aware of the fact that a successful participation in school is closely connected to the personal background of students. Both immigrant children and children with lower socioeconomic status turned out to achieve lower overall results. Heinze et al. (2011) present results from a follow-up investigation of children with migration background using the DEMAT testing material; it was shown that language proficiency in the German language has a higher influence on achievement in mathematics and cognitive performance in general than on reading skills.

If the language used in the educational system is not the speaker's mother tongue, special problems are encountered. In order to account for the specific challenges for second language learners, Cummins (2008) introduced the notions of BICS and CALP in order to distinguish between different levels of language. BICS stands for basic interpersonal communication skills, CALP means cognitive academic language proficiency. Duarte (2011, p.60) pointed to different results showing that the acquisition of BICS can be achieved within two years of being exposed to the new language, CALP abilities usually require at least five years.

In general, academic language shows the characteristics Bernstein pointed out for vertical discourse, whereas everyday speech can be compared to horizontal discourse. Duarte (2011) has given a concise overview on the characteristics of academic language compared to those of everyday speech. The features of the different language levels are listed in Table 1.

Academic language	Everyday speech
Orientation towards written language	Oriented towards spoken language
Abstract, symbolic	Concrete, factual
Context-disembedded	Context-embedded
Generalizing	Specific
Can be technical and domain-specific	Unscientific, general
Linguistically concise	Linguistically diffuse
Precise	Imprecise
Impersonal (uses personal pronouns)	Personal (usually agents are explicit)
High degree of cohesion	Partially unstructured and loose
High lexical density	Low lexical density

Table 1: Main differences between academic language and everyday speech (Duarte, 2011, p. 71, adapted and shortened)

In addition to the obstacles shown in table 1, which are true for the academic register in all languages, Duarte (2011, p.71) lists some features of German academic language. Among these are the use of sophisticated verbs instead of simple verbs with prefixes, adjectival and adverbial attributes, and nominalisations. Gogolin (2009) has introduced the term “Bildungssprache” to account for these special characteristics of German academic language. Referring to Habermas, she defines Bildungssprache as “the language register which enables to gain orientational knowledge by using the means of school education” (Gogolin 2011, p.108, my translation).

Children with migration background in Germany often come from families with a low socioeconomic status and little education (Gogolin 2009, p.267). As shown before, this leads to further disadvantages in the familiarity with elaborated code before entering school. These obstacles are important in all subjects and must be taken into account by teachers.

Language requirements for deductive reasoning

A bigger emphasis on reasoning in the mathematics classroom must take into consideration possible language barriers. Children from all backgrounds are likely to understand language based on basic interpersonal communication skills. On the other hand it is visible in the definition of Bildungssprache by Gogolin given above that academic language abilities are needed in order to gain orientational knowledge in a new context. Thus, fostering CALP should be one aim of education in all subjects, also in mathematics. In addition to this demand, mathematical reasoning has some characteristic features that make academic language not only desirable but also necessary.

Mathematical reasoning takes place *abstracted from concrete situations*. Processes of reasoning in mathematics establish a link between new knowledge and existing knowledge structures. These knowledge structures are internal and show hardly any connection to tangible objects. Furthermore, mathematical reasoning frequently makes use of *generalizing* techniques, especially in the inference rules used in deductions. Another feature of mathematical reasoning is the *precise, coherent and concise form* desired as the outcome of the reasoning process.

All of the named features are also characteristics of the academic language register. From this I conclude that academic language can hardly be avoided in the teaching and learning of reasoning. The required language register can quickly become an obstacle. I am convinced, however, that the close relationship between mathematical reasoning and academic language also offers many learning opportunities.

FIRST INSIGHTS INTO MY RESEARCH WORK

In my research, I am working as a teacher-researcher in a project for children with a migration background whose mother tongue is not German. They come to university once a week to receive support in different subjects in groups of 4-6 learners at no charge. The students come into the project from different schools. From September 2012 until the end of January 2013 I am teaching and researching in two groups of students, one group is in their 9^{th} year in school and the other in their 11^{th} year. There is an option to prolong the research period until July 2013.

I am collecting data from three different sources. Videotaped interviews at the start of the project, before Christmas and towards the end, combined with a reasoning task, are used as control points for the students' views on and abilities in deductive reasoning. In the first interview I found that in the mathematics classes of most of the students, hardly any reasoning takes place. For the development of material and in order to have a second opinion on the developments in the groups, there are weekly consultations with David Reid as an expert on reasoning, which are audiotaped. The third data source is the videotaped material from the lessons. In addition to all that, I keep a research diary in which I keep track of my experiences. The videotaped material will be evaluated at the end of the project, in order to retrace the individual development of mathematical reasoning.

Material development within the project takes place on a weekly basis, constantly taking into consideration the consultations with David Reid and the immediate impressions from the previous lessons. I am developing language sensitive material that creates opportunities for mathematical reasoning. In task creation, I am inspired by the cognitive unity approach developed by Boero et al. (1996) as well as by the approach concerning the local organisation of hypotheses by Hanna and Jahnke (2002). In both groups I detected large gaps in the knowledge from previous school years. This led to the decision of not only focussing on topics from their current grade but also including topics that the children are supposed to have dealt with in the past.

In the following, I will present some material on linear functions from my grade 9 group. In the previous lesson the children had been working with laptops, developing some hypotheses on the influences of the chosen parameters on the slope and y-intercept of linear through a game in Geogebra. In the next lesson I tried to deepen the understanding in this area in a paper task on which the students worked together. Once again, they were given two tasks with points on a coordinate grid; this time, however, the points in each grid belonged to just one function. (Fig. 1 and Fig. 2).

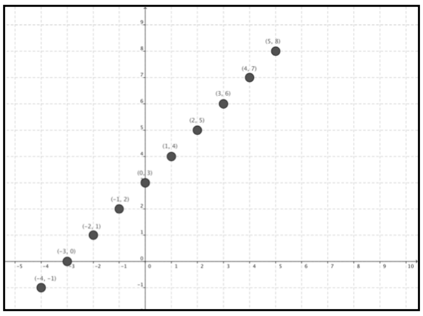

Fig. 1 First task

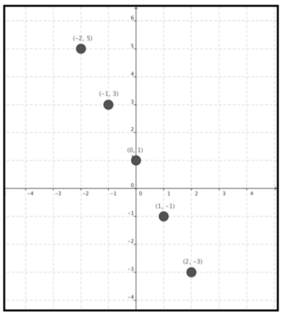

Fig. 2 Second task

Additionally, a table of values for each of the two functions was given. The students had to complete several sentences such as: “If the value for x increases by 1, …”, “The intersection point of graph and y-axis is…”, “If you put a 0 for x into the equation y=m·x+b, the equation simplifies into…”. These sentences make use of language from the academic register. When the students had come up with a suggestion for an equation on which they all agreed, they were allowed to check its correctness on Geogebra.

After having discovered and discussed about the influence of the chosen parameters in linear functions, a third task was given, “Find a linear function which goes through the point (0|2) and is parallel to the function from task two”. The students engaged in a vivid discussion on the solution to this task. There was no agreement on a solution until one girl came up with the argument that m defines the steepness of the graph, and therefore it has to be the same in the two functions.

Eight weeks into the project, a positive attitude towards reasoning and challenging the rationale of statements can be observed in both groups. The students differ highly in their language proficiency, especially among the students from grade 11. Further data and a deeper analyses are necessary to identify specific problem areas. This is one aim of the further analyses that will be conducted after the end of the project.

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