DIVERSITY, DIALOGISM AND MATHEMATICS LEARNING:
SOCIAL REPRESENTATIONS IN ACTION

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Learning is a dialogical process in which diversity plays a major role. This research illuminates the connections between collaborative work, social representations and mathematics learning. We assumed an interpretative approach and developed an action-research project. The participants were these 8th graders (N=21), a teacher/ researcher and two other observers. Data was collected through an instrument to evaluate students' abilities and competencies, tasks inspired in projective techniques, questionnaires, observation, informal conversations and students' protocols. We did a narrative content analysis from which inductive categories emerged. Paula's life trajectory of participation is a paradigmatic example we discuss through the analysis of her social representations and her performances in some mathematics activities.

INTRODUCTION

Freire (2003) stated that education is a political act, based on social interactions and participation, or lack of possibilities to participate. Thus, it is dialectical and dialogical (Marková, 2005), as there are always conflicts that need to be addressed. However, according to dialogism, conflicts contribute to knowledge and progress. It should celebrate and respect students' values and cultures, facilitating their participation in school activities, particularly through their (cultural and social) practices, and the way teachers interpret curricula and put them into practice and the social interactions they promote while solving mathematics activities (César, 2009, 2012). In order to express and respect each student's diversity, allowing them to have access to achievement, social interactions must contribute to the distribution of power, i.e., to students' empowerment (Apple, 1995). As Alrø, Skovsmose and Valero (2005), we assume diversity as a resource for learning as it brings into classes different experiences and perspectives related with diverse knowledge, ways of thinking, solving strategies, abilities, and competencies. Diversity is connected to the participation in different cultures, and the use of different symbolic resources, cultural tools and artefacts (Vygotsky, 1934/1962). Thus, in multicultural settings being aware of the importance of culture, social interactions, and diversity is even more important, as they shape – and are shaped by – students' (mathematics) performances (César, 2009; Cobb & Hodge, 2007; Machado & César, 2012).

Some educational policy documents highlight the need of taking (cultural) diversity into account in order to promote equity and achievement in mathematics (Abrantes, Serrazina, & Oliveira, 1999; NCTM; 2007; Ponte et al., 2007). Learning meaningful mathematics and being able to internalise knowledge after attributing it a sense

(Bakhtin, 1929/1981) is a way to develop some abilities and competencies, which are powerful to avoid exclusion, as they are needed in a society that is increasingly complex and technical. Intercultural and inclusive practices are essential in order to allow students who participate in vulnerable minorities to become legitimate participants while developing mathematics activities, particularly in formal educational scenarios, like school (César, 2009, 2012; Lave & Wenger, 1991; Machado, 2008). In Portugal mathematics assumes an important role in a new construct coined by César (2012): students' life trajectory of participation, in and outside schools. The construct of trajectory stresses the time dimension in learning – and in life – as it includes past, present and future, i.e., direction, path. The trajectories are connected to participation, or the lack of opportunities to participate, i.e., the lack of power. Finally, as César (2012) assumes, learning is a lifelong process. Thus, these trajectories of participation are life trajectories. They begin when we are born and only end up when we die. For teenagers, school, family and their peers groups play a main role in their life trajectories of participation.

Students often experience feelings of rejection, frustration, and construct negative social representations about mathematics that shape their performances in this subject (Machado, 2008; Piscarreta, 2002; Ramos, 2003). Knowing social representations is essential in order to promote students' access to mathematics achievement and to avoid one of the most expressive forms of exclusion: dropping out of school (Abreu & Gorgorió, 2007; Machado & César, 2012). According to these statements, the problem that originated this study is students' negative social representations about mathematics and their impacts in their (under)achievement and in their school and social inclusion. The research questions we address are: (1) Which are these 8th graders' social representations about mathematics at the beginning of the school year?; (2) Which changes do we observe in those social representations during the school year?; and (3) What are the impacts of collaborative work on students' social representations about mathematics, in their mathematics knowledge appropriation and in their mobilization and development of abilities and competencies?

THEORETICAL FRAMEWORK

Social representations are a dynamic, multi-faced and dialogic construct (Marková, 2005; Moscovici, 2000). They give us powerful insights about how the other interprets the contexts, scenarios and situations that s/he is experiencing. Thus, social representations play an essential role in mathematics learning process, particularly in formal educational scenarios (Abreu & Gorgorió, 2007; Machado & César, 2012). Social representations are constructed since we begin interacting with other people and they are shaped by our life experiences but also by the *media* or the values of a particular culture. As many students develop negative social representations about mathematics it is important to develop classroom practices that facilitate changing these negative social representations into more positive ones. It means that teachers should create spaces and times that facilitate students' transitions between the different cultures in which they participate (César, 2009), like the school culture and

their home culture(s), facilitating students' mobilisation of abilities and competencies, particularly in mathematics classes.

Teachers should also reflect upon the importance of power relations (Apple, 1995) and they should distribute power among them, the students and other educational agents (César, 2009). Similar to the appropriation of knowledge, the internalisation of empowerment mechanisms happens on the inter-individual plane and the intra-individual plane. Accordingly, César (2012) coined two new constructs: inter- and intra-empowerment mechanisms. She states that there are inter-empowerment mechanisms, used by those who have more power. Later, students are able to internalise them and they become intra-empowerment mechanisms. She illuminates the importance of these inter- and intra-empowerment mechanisms in students' mathematics performances and in their life trajectories of participation, particularly for those whose cultures are more distant from the school culture. Perret-Clermont (2004) designates as thinking spaces those where students can interact dialogically sharing their ways of thinking, solving strategies, doubts and fears. They are also assumed as a security and trust space and time, where students (inter)act as legitimate participants and power is distributed (Apple, 1995; César, 2009, 2012). This aspect is very important because if we want to develop a critical mathematics education (Alrø, Ravn, & Valero, 2010) and equity among students (Cobb & Hodge, 2007), we allow them to assume their own voices (Wertsch, 1991), instead of keeping some of them silenced, particularly in classes.

In multicultural settings in which diversity is the main tone, collaborative work, particularly in dyads and small groups, plays an essential role (César, 2009; Machado, 2008; Tielman, den Brok, Bolhuis, & Vallejo, 2012). In order to develop collaborative practices one needs to negotiate a coherent didactic contract that promotes autonomy, accountability, critical sense, respect for each other's cultures, and focus on students' ways of acting and reacting. Thus, the teachers' practices change and teachers face much more challenges, as they have to work outside their comfort zones (Skovsmose, 2000). They should act like a mediator rather than someone who possesses all the knowledge or, as Papert (2001) claims, they should act as co-learners. Then, they need to elaborate, adapt, and/or select tasks with different natures regarding students' characteristics, interests, and needs. Besides that, teachers should give working instructions that facilitate students' engagement in mathematics activities while working in their zone of proximal development (ZPD) (Vygotsky, 1934/1962) and respecting their own cultural mental tools (César, 2009, 2012). This also means reflecting upon the evaluation system and using diverse means of evaluation, conceiving this process as an auto-regulatory mechanism regarding meaningful learning.

METHOD

This study is part of a Master thesis developed within the *Interaction and Knowledge* (IK) project, which formally lasted 12 years (1994/95-2005/06). IK main aims were

studying and promoting collaborative work in formal educational scenarios, and promoting a more inclusive, intercultural and high quality mathematics education (César, 2009). This project developed three research designs: (1) *quasi-experimental* studies; (2) action-research projects; and (3) case studies (for more details see César, 2009). We assumed an interpretative approach (Denzin, 2002) and developed an action-research project (Mason, 2002) at a very multicultural school near Lisbon whose region is affected by unemployment and poverty. The participants were the students (8^{th} graders, N=21), the teacher/researcher and two other observers. Students worked collaboratively during the whole school year, mainly in dyads but also in small groups (e.g., during the project work, in Statistics or in Functions). Data was collected through questionnaires (Q) (at the beginning and at the end of the school year), tasks inspired in projective techniques (TIPT) (at the beginning of the 1^{st} and 2^{nd} terms and at the end of the 3^{rd} term), an instrument to evaluate students' abilities and competencies (IACC) (first week of the school year), participant observation (written in the researchers' diary - D) and students' protocols (collected during the whole school year). Such data was then treated and analysed through a narrative content analysis (Clandinin & Connelly, 1998), performed in a successive and in-depth way. Inductive categories emerged from this analysis (for more details see Machado, 2008) that allowed to trace these students' life trajectories of participation, in and outside school, as well as to illuminate the inter- and intra-empowerment mechanisms that facilitated students' mathematics learning (César, 2012).

RESULTS

According to the epistemological and pedagogical principles of the IK project the practices we develop included a different type of practices since the first week of the school year. In that week teachers do not teach any mathematics contents. They use three instruments (TIPT 1, Q1, IACC) to have access to a deeper knowledge about students' characteristics, interests, and needs. The TIPT 1 allows teachers to know the students' social representations about mathematics. The Q1 gives information regarding students' lives (about parents or other relatives, hobbies, future job, among other aspects), and their life trajectory of participation in school. The IACC allows knowing the abilities and competencies students are able to mobilise and the ones they need to develop. This information is very important for teachers to develop adequate practices (e.g., nature of the tasks and working instructions) regarding students' cultural diversity, i.e., respecting their own characteristics, avoiding deficit interpretations, assuming that learning is a lifelong process, as well as development. This means that teachers' practices should contribute to promote students' knowledge and development, even when the educational system needs improvements. Assuming collaborative practices, based on dialogical interactions, also allow teachers to promote (academic) positive self-esteem and to avoid (subtle) ways of rejection of mathematics and forms of exclusion.

We are going to analyse a paradigmatic example (Paula, pseudonym) that illuminates other students' life trajectories of participation during that school year. She was 14

and lived in a low socio-economical area. She and her family were Portuguese but she had characteristics that put her away from the mainstream culture: she did not have a computer at home and did not spend time in social networks, she did not wear trademark clothes, and she did not have any sort of gadgets that were usual for many Portuguese teenagers. Still, she was similar to many other students in her school and she felt accepted by her peers. The expectations her family had towards schooling were low, as they did not believe a school diploma would provide for her living in the future. Thus, she did not put any effort or trust in school and in her performances, particularly in mathematics. She was polite, but did not engage in school activities. Thus, she experienced subtle forms of social and school exclusion. Her socialisation among her peers included extra-school activities such as going to the cinema, to the shopping centre, to the esplanades, or to the gardens near her school. Thus, she had the same kind of life experiences as her classmates.

Paula was doubling the 8th grade. Her social representation about mathematics was a very common one: a difficult and boring subject. As she reports: "It is the subject in which I have more difficulty and I think it's less interesting" (Paula, Q1, September 19th, 2006). She also added, "(…) [mathematics] is tiring" (Paula, Q1, September 19th, 2006). This illuminates a strong rejection and a negative self-esteem regarding this subject probably shaped by her previous school experiences.

> For me mathematics is a subject in which a lot of attention is needed and a teacher who knows how to explain it well and who doesn't transform mathematics into something even more boring (*mais secante*) than it already is; and it is also a set of themes that include numbers. (Paula, TIPT 1, September 19th, 2006)

She constructed a negative social representation about mathematics because she associated it with a boring activity and something that needs a lot of attention. She also related mathematics to numbers. Another important feature of her answer is that she never describes mathematics as important for daily life or to get a better job, as stressed by many students (Machado, 2008; Piscarreta, 2002; Ramos, 2003) and by the *media*. This was probably shaped by the low importance of school in her life trajectory of participation and by the lack of intra-empowerment mechanisms (César, 2012). She participated in a vulnerable culture with different life goals, ways of thinking about their future, and their role in the society. For her and her family, poor people only went to school because it was compulsory. Their lives were not going to be improved by school knowledge and studying many hours was a waste of time.

When we developed collaborative practices our aim was to support students' (mathematics) knowledge appropriation and their mobilization and development of abilities and competencies. This was our way of avoiding exclusion, of promoting equity and inclusion. In order to achieve this we needed to promote inter-empowerment mechanisms that facilitate students' access to achievement, particularly in a school system that was not tailored for them. Our commitment was precisely to avoid exclusion and to provide these students with a high quality education.

The majority of these students did not study in a regular basis outside school. One of the practices we developed to improve students' study was our conception of the homework: (1) they received some homework to do once a week, always in the same weekday; (2) what counted for the evaluation was doing or not doing the homework; and (3) if they were not able to do it or to finish it they had to explain why that happened. At least once a term the homework needed the collaboration of family and/or friends in order to collect data needed to solve it. This was one of the inter-empowerment mechanisms we put into practice in order to promote an intercultural education, that valued family's contributions to school learning. Different solving strategies used to solve the homework were discussed in class. This facilitated students self-regulation of their study, and facilitated the internalisation of inter-empowerment mechanisms as students realised we were valuing their contributions. It also promoted respect towards diversity and improved students' (academic) positive self-esteem – essential steps towards more positive social representations about mathematics and about themselves as mathematics learners.

During the first term (September to December) Paula only solved one in seven homework. As she stated in an informal conversation registered in the researcher's diary, in the first weeks of the school year she preferred "Not doing it rather than being confronted with another failure" (D, September/October, 2006). After some talks with her teacher and observing what was going on with her colleagues she decided to do her first homework (see Figure 1).

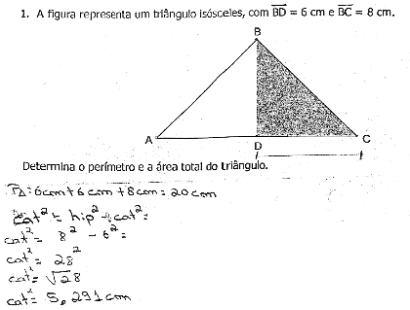

Figure 1: Paula solving strategy in her homework (October 19th, 2006)

The task was composed by two parts, which included calculating the perimeter and the area of the triangle. As we can observe in Paula's solving strategy she was not able to mobilise the knowledge related to the Pythagoras theorem. Thus, although using it to calculate half of the base of the triangle, she did not realize that what she did to calculate the perimeter was wrong. As some other students also used this solving strategy, the teacher decided to discuss it when he gave the homework feedback to the students. He started by asking Paula to explain her reasoning. She went to the blackboard and drew the triangle. She explained that she calculated first the measure of DC and then the perimeter. However, as soon as teacher asked why did she calculate the measure of DC if she only used two values (8 and 6), she said: "Oh sorry! I am so dumb! I thought wrongly!" (D, October 19th, 2006). Then, she explained what it should had been done and with other students' help she was able to

solve the task. In that term Paula only did this homework. In the following terms she did the majority of them (2nd term - four in six; 3rd term - two in three), which illuminates that the changing process regarding participation is slow, particularly regarding outside school mathematics activities, i.e., the ones in which students feel more insecure. Thus, it takes time to become part of students' life trajectory of participation. Once the teacher was able to empower Paula, particularly through the way he acted in class, promoting inter-empowerment mechanisms, she progressively became more engaged in mathematics activities and then she was able to internalise and use intra-empowerment mechanisms (César, 2012).

The next task (see Figure 2) was connected with the previous one but also with some daily life situations. It is composed by two questions in which students: (1) have to calculate the area occupied by the four triangular faces in the figure; and (2) have to calculate the cost of ink cans knowing that 1 litre of ink costs 1,59 Euros and 2 litres of ink allow painting $10m^2$. Paula and Carolina started to do the first question in which Paula took the leadership. They calculated all the data they needed.

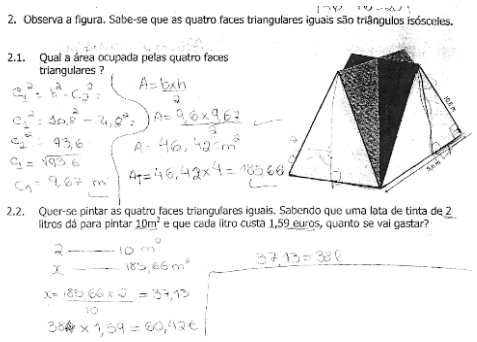

Figure 2: Paula and Carolina solving strategy (October 26th, 2006)

In the second part Paula did not know what she was meant to do. She was confused by so many data. Thus, Carolina took the leadership and explained Paula what was the solving strategy that she suggested. This became necessary, as the students knew that in the whole-class discussion, all students could be asked to explain the solving strategies of their dyad. When Carolina did the last step – to calculate how much was the cost – Paula interrupted her and said that she was doing a mistake, because in a store we do not buy 37,13 litres. We buy 37 litres or 38 litres (D, October 26th, 2006). After Paula's comment, this dyad finished the second question. This episode illuminates that the role of the most competent peer, while working in their ZPD (Vygotsky, 1934/1962), may change during a school activity (Paula, Carolina, Paula). This is important if we want students to become more confident, autonomous, and developing a positive self-esteem. It also illuminates that Paula was able to connect a daily life situation with the school mathematics, i.e., she was able to do what Abreu, Bishop and Presmeg (2002) called transitions between two contexts and cultures (home and school). This is particularly important for students whose home culture(s) is/are more distant from the school culture, and for those who value school knowledge less regarding their future.

At the end of school year, Paula and her colleagues answered TIPT 3. Analysing Paula's drawing (see Figure 3) we observe that she changed her social representation. Several aspects illuminate this change: (1) the teacher has a happy face and he is closer to the students than to the blackboard. Moreover, what the teacher is saying is understood by all the students; (2) there are not any chairs and tables, which illuminates the freedom of moves students connect to the autonomy collaborative work promotes; (3) the blackboard has a smiling face, i.e., it became a nice place where she felt secure and confident; and (4) the mathematics contents (Pythagoras theorem and special cases of multiplying binomials) are connected with two different experiences: (a) related to the Pythagoras theorem, was when she started to change her social representation about mathematics and started to engage in mathematics activities; and (b) it was the content she disliked the most and in which she experienced more difficulties (Paula, Q2, June 15th, 2007). By then she already realised that it was better trying than not doing anything because otherwise her colleagues and her teacher could not do anything in order to facilitate her knowledge appropriation (D, June 15th, 2007). For Paula, this was a very important step and corresponded to the internalisation of some inter-empowerment mechanisms, using them as intra-empowerment mechanisms in future situations.

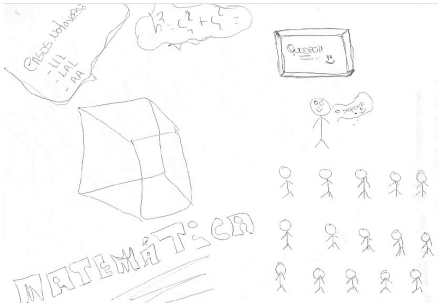

Figure 3: Paula's drawing in TIPT 3 (June 15th, 2007)

This illuminates the importance of teacher's way of acting and reacting, and of the development of inter- and intra-empowerment mechanisms (César, 2012). These mechanisms allow students to become confident and to participate in mathematics activities, feeling that each one has a space and a time to be a legitimate participant (César, 2009, 2012; Lave & Wenger, 1991). The Portuguese educational system has no optional subjects until the 9th grade, and does not fit the needs of many students. Nevertheless, teachers' practices can make a difference and avoid exclusion.

She also chose to write. Her writing corroborates what we observe in her drawing. Paula stated very clearly that by then mathematics was real nice and funny.

> In this term for me mathematics was super, mega, deep, huge, intensely fun… (Paula, TIPT 3, June 15th, 2007).

During that school year, Paula had Level 2 (1st term) and Level 3 (Level 1 is the lowest and Level 5 is the highest mark; you fail if you get Level 1 or 2) in the other two terms. She was able to progress to the 9th grade without any negative marks. She

stated that "The mathematics classes could be always like this… for next school year" (Paula, Q2, June 15th, 2007), which illuminates the important role that collaborative work had in her life trajectory of participation, during that school year. It also illuminates that by then she realised that being at school and learning could be important for her future and to get a better job. Finally, above all she felt much more confident in her abilities and competencies, and she was able to mobilise them in many more contexts and situations, as we observed and she also told us in several informal conversations.

FINAL REMARKS

Dealing with diversity is more challenging and meaningful if we assume it as vehicle for inclusion and learning, namely in mathematics. Teachers need to know students' social representations about mathematics since the beginning of the school year to decide about the tasks and working instructions they will use in classes. Gathered with other information from the IACC, a questionnaire and the observation it shapes decisions about the first dyads. Collaborative work, in dyads and small groups, associated with a coherent didactic contract and working instructions can be used as a meditational tool to facilitate the change of students' social representations. It also facilitates students' engagement in school activities, their mathematics knowledge appropriation and the mobilization and/or development of abilities and competencies, as it promotes dialogical social interactions between students, and also with their teacher. Developing practices that shape more intercultural and inclusive scenarios in which students have voice(s) and the power is distributed facilitates their legitimate participation in mathematics activities, promoting a high quality mathematics education. It also allows students to internalize inter-empowerment mechanisms and then using them as intra-empowerment mechanisms (César, 2012). Thus, their life trajectories of participation are also changed and include more possibilities of choice.

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