THE DEVELOPMENT OF FOUNDATIONAL NUMBER SENSE IN ENGLAND AND HUNGARY: A CASE STUDY COMPARISON

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Foundational number sense - being able to operate flexibly with number and quantity - is a predictor of later mathematical achievement. In this paper, drawing on lessons on number sequences to grade 1 children, we examine how two teachers, one English and one Hungarian, construed locally as effective, created opportunities for children to develop foundational number sense. The Hungarian teacher, in ways typical of that country's mathematics teaching tradition, offered frequent and coherent opportunities for students to develop foundational number sense. The English teacher, working in a tradition whereby interactive technology increasingly mediates classroom discourse, offered few and less coherent opportunities, masked by the teacher's frequent attention to display features of the technology.

INTRODUCTION

Described as a "traditional emphasis in early childhood classrooms" (Casey et al 2004: 169), the quality of young children's number sense is a key predictor of later mathematical success, both in the short (Aunio and Niemivirta, 2010) and the longer term (Aunola et al, 2004). Consequently, particularly as number sense deficits tend to lead to later difficulties (Jordan et al., 2007), the development of children's number sense "is considered internationally to be an important ingredient in mathematics teaching and learning" (Yang and Li 2008, p. 443). Importantly, without appropriate intervention children who start school with limited number sense are likely to remain low achievers throughout their schooling (Aubrey et al., 2006).

Defining foundational number sense

Our reading of the literature reveals two distinct number sense concepts. The first, which we have labelled foundational number sense, concerns the number-related understandings children develop experientially before and during the early years of formal instruction. The second, applied number sense, draws on the first and concerns the number-related understanding necessary for people not only to achieve mathematical success but also function effectively in society. In this paper, while remaining mindful of the latter, we focus on the former.

In broad terms foundational number sense is construed as a child's ability to operate flexibly with number and quantity and comprises, inter alia, seven, cross-culturally constant, components on which our analyses are based. These are:

1. Awareness of the relationship between number and quantity (Berch, 2005; Clarke and Shinn, 2004; Van de Rijt et al., 1999; Griffin, 2004).

2. Understanding of number symbols, vocabulary and meaning (Clarke and Shinn, 2004; Van de Rijt et al., 1999; Malofeeva et al., 2004; Yang and Li, 2008).

3. Systematic counting, including notions of ordinality and cardinality (Gersten et al., 2005; Griffin, 2004; Malofeeva et al., 2004).

4. Awareness of magnitude and comparisons between different magnitudes (Gersten et al., 2005; Griffin, 2004; Ivrendi, 2011; Jordan et al., 2007; Malofeeva et al., 2004).

5. An understanding of different representations of number (Ivrendi, 2011; Jordan et al., 2007; Yang and Li, 2008),

6. Competence with simple arithmetical operations (Berch, 2005; Ivrendi, 2011; Yang and Li, 2008).

7. An awareness of number patterns including recognising missing numbers (Berch, 2005; Clarke and Shinn, 2004; Jordan et al., 2007).

Importantly, "number sense develops gradually over time as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Sood and Jitendra, 2007, p. 146).

DATA COLLECTION AND ANALYSIS

In this paper we compare excerpts from two lessons taught to grade one children in England and Hungary. Both excerpts were focused on number sequences and their role in the development of more general number competence. The lessons from which they were drawn were each part of a wider collection of videotaped lessons gathered independently of each other. The English lesson derived from the second author's PhD case study examination of primary mathematics teachers' enactment of whole class teaching. The Hungarian lesson derived from a study of exemplary Hungarian primary mathematics teaching undertaken by the first author to inform curriculum development activities in England. Both teachers were construed against local criteria as effective in the manner of the Mathematics Education Traditions of Europe project (Andrews, 2007). Both teachers, with microphones, were video-recorded in ways that would optimise the capture of their actions and utterances. Both data sets entailed repeated observations of a small number of case study teachers over a period of several months in order to ensure a sense of the typical lesson. The Hungarian lessons were supported by a home-based English-speaking colleague providing a contemporary translation augmented by the first author's sufficient understanding of Hungarian to be able to follow much of the discourse of a mathematics classroom. In other words, while the two sets were collected independently, they were gathered in similar ways and amenable to similar analyses. With respect to analysis, the same approach was used for both excerpts. Each was viewed simultaneously and repeatedly by all three researchers. With repeated viewings it became clear to all of us not only that each excerpt comprised three distinct phases, which frame our analyses, but also which components of foundational number sense were addressed at any given time.

RESULTS: THE ENGLISH EXCERPT

This excerpt derives from a lesson taught to 6 and 7 year olds on number sequences and patterns. It began with the teacher, Sarah, reminding her children about previous work on sequences, how odd and even numbers create an alternating pattern in the number system, and informing them that today they would be looking at some more sequences and number patterns. The 5 x 10 grid below was displayed on the interactive white board – a resource typical of English classrooms - with the cells containing the first six even numbers shaded. Sarah asked if anyone could explain the pattern. One child responded by saying that they all end in 0, 2, 4, 6 or 8, after which Sarah commented that they are all even. Next she asked about the column patterns and, after a suggestion that the numbers in each column end in the same digit, Sarah accepted the suggestion that the column pattern goes odd even odd. At this point, exploiting the whiteboard's software, she displayed the odd columns in one colour and the even in another.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	33	24	25	26	27	28	29	30
31	32	43	34	45	36	37	38	39	40
41	42	53	44	55	46	47	48	49	50

Commentary

In these first few minutes a number of number sense categories appeared to have been addressed. For example, Sarah's explicit emphases seemed very much geared towards inducting her children into an awareness of number patterns alongside a clear expectation that children recognise number symbols and vocabulary.

Next, Sarah put up another 5 x 10 grid but with no coloured cells. Beneath the grid was the following:

1, 4, 7, 10, 13, __, __

She asked how other numbers in the sequence could be found and invited her class to look at the number grid. Having evoked no responses Sarah tapped, in turn, each of the five numbers to change the colour of their cells and create the image below. At this point she commented that the grid looked different from that of the previous problem and asked *have I done it wrong?* Different students offered, tentatively, both negative and positive responses, which went without comment.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	33	24	25	26	27	28	29	30
31	32	43	34	45	36	37	38	39	40
41	42	53	44	55	46	47	48	49	50

Next, Luke raised his hand and the following ensued:

Sarah	Luke?
Luke	Sixteen
Sarah	Why sixteen?
Luke	Because you're adding on three
Sarah	Because it's adding on three isn't it (she taps the cell to change the colour)... What's going to be the next one? Isla?
Isla	Nineteen
Sarah	What's going to be the next one? Ian?
Ian	Twenty-two
Sarah	Twenty-two (she taps the cell). And the next one? (more hands go up this time) Rachel?
Rachel	Twenty-five

Commentary

In the above can be seen further evidence of different aspects of foundational number sense. The four students' were clearly extending the sequence given them. Also, the assertion that they were adding on three indicated an engagement with simple arithmetical operations and, of course, they were still being made aware of the relationship between symbols, their vocabulary and meaning.

By now the grid looked as below. The following ensued:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	33	24	25	26	27	28	29	30
31	32	43	34	45	36	37	38	39	40
41	42	53	44	55	46	47	48	49	50

Sarah	Twenty-five (she clicks on the cell, which becomes red) Can anybody see any colour patterns coming out of this?
Boy	It looks like a bit like stairs!
Sarah	Rosie? (Rosie says something inaudible so Sarah walks to the middle of the room to hear what Rosie says)
Sarah	Yes, they're going diagonally aren't they. Yes when we extended this pattern we started to see that. There's a diagonal pattern made by the squares coloured in. Now, if Rosie's right then this one's (she taps 34, which turns red) going to be in our sequence. I'm going to fill it in and then count on three each time. One, two, three (she taps 28, which change it to

	red). One, two, three (she taps on 31). One, two, … (she points to 34 which is already changed to red and turns to face the children)
Children	Threeee (a few children shout out)

Commentary

In this closing episode Sarah appeared to be encouraging systematic counting from an ordinal perspective as well as further opportunities for children to recognise number symbols, vocabulary and meaning. Thus, over the whole excerpt, Sarah seemed to have addressed elements of four of the seven categories of foundational number sense.

RESULTS: THE HUNGARIAN EXCERPT

This excerpt was taken from a sequence of lessons focused on children's coming to know and work with integers to 20. The lesson began with Klara, having written on the board prior to the start of the lesson, the configuration shown below.

–	–	–	–	–	–	–	–	–
3	7	6	10	_	_	_	_	_

Having been invited to do so, the class read out the numbers in unison as Klara pointed to each in turn. Next, moving from left to right, she invited volunteers to explain how each number could be derived from the one preceding it. Students volunteered that the first operation was add four, followed by subtract one and add four. With each offering Klara wrote the operation underneath, as shown below, before inviting predictions as to what operation would be expected next. Eventually, after several contributions, the table was completed.

–	–	–	–	–	–	–	–	–
3	7	6	10	_	_	_	_	_
+4	-1	+4						

Commentary

During this period it seems to us that Klara had encouraged several aspects of foundational number sense. Firstly, an introduction during which children were invited to recognise and read the numbers on the board was clearly focused on the recognition of number symbols and their vocabulary. Secondly, in the ways in which successive numbers were identified, Klara was addressing simple arithmetical operations. Thirdly, the episode was explicitly focused on an understanding of number patterns.

On completion of this first task, Klara showed the class a set of cards, each of which had a letter written on it. She announced that they were going to play a game, which involved her asking questions to which the answer would be one of the numbers in the sequence. Each correct answer would yield a letter, as a reward, that will eventually spell out a word that would tell the class where it would be going in the story of this particular day. The following reflects the first minute of the next five minutes of discourse.

Klara	So my first statement is… please look only at the numbers on the board… I am thinking of the largest one-digit number. Balasz?
Balasz	Six (Pupils protest)
Klara	Look at the sequence again, and please, correct yourself.
Balasz	Seven
Klara	Look at the number line… Ferenc?
Ference	Nine
Klara	That's right. So I will give you a reward for the nine (Klara attaches a card, showing the letter Í, to the board above the number nine). When you know the solution, please keep it secret… The next number I am thinking of… You mustn't look behind you (Referring to a picture on the back wall) is the value of the black stick in our collection. Perszi?
Perszi	Eight (Pupils protest)
Klara	(to Perszi) Look around, the others don't agree with you… Mara?
Mara	Seven
Klara	Let's see who's correct. (They all look at the back wall, where they can see the members of the Cuisenaire rod collection and their values)
All	(In chorus) Mara was correct (at which point Klara places a card with the letter Á above the seven).

The lesson continued with Klara asking a different form of question for each of the numbers in the sequence. These included statements like, a number two smaller than nine, the largest two digit number, the smallest one digit number, a number whose digits add up to 4, and so on. In each case at least one child was involved in publicly responding to the questions posed and with each correct response a new letter was attached to the board. Eventually, as shown in figure 4, the following emerged with only the number ten left without its corresponding letter.

B	Á	B	_	Í	N	H	Á	Z
3	7	6	10	9	13	12	16	15

+4 -1 +4 -1 +4 -1 +4 -1

Commentary

During this phase of the episode several number sense categories were addressed. In considering the largest one-digit number Klara encouraged not only recognition of number symbols, their vocabulary and meaning but also awareness and comparisons of magnitude. The statement concerning a Cuisenaire rod and its numerical value addressed not only an understanding of different representations of number but also an awareness of the relationship between numbers and quantities. Every statement could be construed as focusing on different representations of number in the sense, for example, that a number two smaller than nine is a different representation of seven. Similarly, several statements involved simple arithmetical operations.

Having identified all the letters bar the one linked to ten Klara passed responsibility to her students and invited them to offer statements appropriate to that number. This led to the following:

Mara:	It is the bigger neighbour of the number 9.
Ildikó:	It is the smallest 2-digit number.
Csaba:	It is the smaller neighbour of the number 11.
Gabor:	The sum of its digits is 1.
Judit:	Even number.
Klara	Have we got anything else?
Zsolt:	It is the sum of the 1 and 9.
Klara:	Yes, the sum of the 1 and 9… and who knows the letter in my hand?
All:	Sz (the juxtaposition of s and z in this manner is, in Hungarian, an alphabetic letter with a sound similar to the s in sun)
Klara:	Yes, and where are we going today?
All:	Bábszínház. (Puppet theatre)

Commentary

In this final episode was further evidence of Klara's promotion of number sense. For example, both understanding of different representations of number and recognition of number symbols, their vocabulary and meaning was, we believe, implicit in all the contributions. Simple arithmetic could be seen in Gabor's statements that the sum of ten's digits is one and Zsolt's suggestion that ten is the sum of one and nine. Awareness of number patterns was implicit in Judit's even number suggestion, while awareness of magnitude was implicit in Mara's bigger neighbour of nine and Eva's smaller neighbour of eleven.

DISCUSSION

In this paper we have attempted to show how opportunities for students to acquire foundational number sense played out in two culturally different classrooms. In neither case was the teacher focusing intentionally on the development of foundational number sense as both were addressing, albeit in different ways, the mathematics of number sequences. However, through an analysis of such snapshots we can examine the extent to which the development of foundational number sense is an integral, albeit implicit, component of children’s early school experiences of mathematics. Moreover, since both teachers were construed locally as effective, such snapshots may offer insight into how teachers, in different cultural contexts, have been conditioned - by their experiences as learners of mathematics, their professional training and subsequent career opportunities - to address such issues.

The analyses above, summarised in table 1, indicate both similarities and differences in the ways in which foundational number sense was addressed. In respect of similarities both teachers addressed several categories, with Klara addressing six of the seven categories and Sarah four. Both encouraged, throughout their respective excerpts, students’ recognition of number symbols, vocabulary and meaning. Both encouraged the awareness of number patterns and missing numbers and both exploited simple arithmetical operations. In respect of differences Klara addressed three categories, the relationship between numbers and quantities, comparisons of magnitude and different representations of number that Sarah did not, while Sarah was seen to address systematic counting when Klara did not.

	Sarah’s episodes			Klara’s episodes		
Relationships between numbers and quantities					X	
Number symbols, vocabulary and meaning	X	X	X	X	X	X
Systematic counting			X			
Comparisons of magnitude					X	X
Different representations of number					X	X
Simple arithmetical operations		X		X	X	
Number patterns and missing numbers	X	X		X	X	

Table 1: the distribution of the categories across the excerpts’ episodes

However, while it is clear that both teachers encouraged various aspects of foundational number sense as part of the incidental learning (Radwan, 2005) that took place in their lessons it seems to us that Klara’s was a more didactically complex encouragement than Sarah’s. Put crudely, Klara addressed, on average, four categories of foundational number sense per episode while Sarah addressed barely two. Klara presented opportunities to reason mathematically, while Sarah seemed to subordinate such reasoning to an examination of the coloured patterns on the interactive whiteboard. This latter expectation, it seems to us, seemed not only a distraction from children’s learning of mathematics but indicative of a more general concern that the integration of such technologies in English schools has created ambiguity of purpose for English teachers. Moreover, if number sense develops gradually as a result of exploring and visualizing numbers in different contexts (Sood and Jitendra, 2007) then Klara’s practice seems more likely to succeed than Sarah’s.

Interestingly, both teachers' practices find some resonance with earlier studies of mathematics teaching in the two countries, albeit at the level of the upper primary classroom rather than the lower primary. For example, Andrews (2009) found Hungarian teachers exhibiting didactical sophistication in their encouragement of cognitively demanding but coherent learning outcomes. The same study found English teachers exhibiting relatively unsophisticated didactical practices in their promotion of substantially more modest and less coherent goals. That is both Klara's and Sarah's practices appeared commensurate with that of their compatriot colleagues working in later phases of schooling. Thus, the limited evidence of this study indicates that teachers in the first years of schooling, teachers defined locally as effective, behave in ways similar to their compatriots and that the Hungarian tradition seems more likely to facilitate a secure foundational number sense than the English. That being said, clearly more research in this particular field is necessary if we are to understand more fully how teachers induct their learners into this essential mathematical prerequisite.

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