

FACETS OF THE PRESENTATION OF THE CARTESIAN COORDINATE SYSTEM IN EULER'S *INTRODUCTIO IN ANALYSIN INFINITORUM* AND LACROIX'S TEXTBOOKS¹

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*This paper studies the presentation of the Cartesian coordinate system in Euler's *Introductio in Analysin Infinitorum* and in Lacroix's *Traité du calcul différentiel et du calcul intégral* and *Traité Élémentaire de Trigonométrie Rectiligne et Sphérique, et d'Application de l'Algèbre a la Géométrie*, searching for what components made possible its systematization, and bearing in mind students' difficulties.*

INTRODUCTION

It is a well known fact that students have difficulties with understanding and dealing with the representation of functions in the Cartesian coordinate system (CCS). This didactic problematics has led us to determine “which texts must be sought out in history and what questions we should address to them” (Puig, 2011, p. 29). The texts chosen are Euler's *Introductio in Analysin Infinitorum* (1748), and its French translation from 1796-1797, and Lacroix's *Traité du calcul différentiel et du calcul intégral* (1797) and *Traité Élémentaire de Trigonométrie Rectiligne et Sphérique, et d'application de l'Algèbre a la Géométrie* (1797).

REASONS TO CHOOSE THE TEXTS

The reasons to choose Euler's *Introductio* and Lacroix's textbooks are, first, that we wanted to study texts from the moment in history when the present way of representing functions in Cartesian coordinates was being constituted, and from the moment when it was being incorporated as a teaching topic in textbooks. Next, the main reason to choose Euler's *Introductio* is that it is one of the first books to deal with Cartesian coordinates in a systematic way. Lacroix's textbooks have been chosen because 1) they elementarize mathematics in order to teach it (Schubring, 1987), 2) they deal specifically with Cartesian coordinates in a progressive way, and 3) they had a big impact in the teaching of mathematics, not only in France, but also in Spain. Lacroix's *Traité Élémentaire de Trigonométrie Rectiligne et Sphérique, et d'application de l'Algèbre a la Géométrie* was translated into Spanish, as part of the *Curso completo elemental de matemáticas*, a Spanish translation of Lacroix's textbooks. This translation was widely used because it was established as an official textbook by King Fernando VII's law-ranking decree of 1824, whose Article 42 states “in all these Chairs [referring to the Chairs of Mathematics and Sciences in

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Universities], lessons will last one hour and a half in the morning, and one hour in the evening; being used as textbook the Pure Mathematics by Mr. Lacroix translated by Rebollo”. As far as we have found, the Spanish translation *Tratado elemental de trigonometría rectilínea y de la aplicación del álgebra a la geometría* was published eight times, the 8th edition being from 1846. We have used the Spanish 6th edition of 1820 (Lacroix, 1820), and the original French 4th edition (Lacroix, 1807). No Spanish translation of Lacroix’s *Traité de calcul* was published. We have used the original French first edition (Lacroix, 1797).

COMPONENTS OF THE CONSTITUTION OF CCS

As a result of our study, we state that the main components that make possible the systematization of Cartesian coordinates as presented in these texts are:

1. The endowment of negative quantities with meaning both in algebra and geometry, and the setting of a fixed origin (of coordinates).
2. The constitution of the concept of abscise.
3. The movement from the notion of applicate to the ordinate concept.
4. The movement from coordinates as segments to coordinates as distances, and the consequent movement to coordinates as numbers.
- 5 The establishment of absolute coordinate axes, i. e., axes not specific to the curve.

A study of student difficulties in the light of these components is in progress.

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