TEACHING MODULES IN HISTORY OF MATHEMATICS TO ENHANCE YOUNG CHILDREN'S NUMBER SENSE

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This study aimed to investigate the potential effects of teaching modules in history of mathematics in order to develop number sense in young children aged 5. We reanimated two kinds of historical artifacts as using tally marks and pebbles for counting activities within symbolic play context. Data were obtained by anecdotal records during the implementation of the modules at the Fall of 2012-2013 academic year. Data analysis was made through Dunphy's framework for number sense in young children. Findings revealed that the modules could contribute to various aspects of number sense. We finalized with discussion on the findings and implications for further research on using history of mathematics with younger children.

INTRODUCTION AND THEORETICAL FRAMEWORK

Number is one of the main concepts for primary mathematical reasoning in young children (Cooke, 2005). Children exhibit different kinds of behaviours and skills as a sign of their development in number sense. According to Charlesworth, Lind and Fleege (2003), children are able to relate numbers of objects with counting, to compare the number of objects, and to master the benchmarks as five and ten. The children aged between six and eight can make one-to-one correspondence, meaningfully count, and notice the cardinal and ordinal numbers in a set of objects (National Council of Teachers of Mathematics [NCTM], 2000). The most basic skill may be rational counting, which requires mastering one-to-one correspondence, arranging the names of numbers in exact sequence, being able to continue counting from a predetermined number, and perceiving total amount of objects in a set (Reys, Lindquist, Lambdin, Smith, & Suydam, 2001). Children are initially able to transfer their knowledge of rote counting into rational counting skills in four years of age (Charlesworth et al., 2003). Another issue is that children should be aware of numbers within verbal and written communication as a part of social life in the community (Tolchinsky, 2003). Indeed, it is possible for them to initiate and improve number sense through daily life experiences (Dunphy, 2006b) such as pulling on one glove for each of the hands before starting to play snowball. At this period, structured number activities also may be employed such as using computer software in which touching with fingers simultaneously creates visual objects (Ladel & Kortenkamp, 2011). Such experiences should be supported by parents and early childhood practitioners since communication with relatively intellectual people builds on children's learning (Vygotsky, 1978).

Young children enrol in primary school at the age of six in Turkey. Thus, early childhood education given before this age seems to be crucial in order to develop

number sense for a meaningful learning of mathematics concepts in primary school. For this purpose, relevant objectives are set in Turkish pre-school curriculum (Ministry of National Education [MoNE], 2012) as to count the objects given in a set, to rhythmically count on and back, to determine ordinal numbers in a set, to group objects with specific benchmarks such as five, and to make one-to-one correspondence between objects. Numbers up to 20 are used in the classroom activities. The curriculum also emphasized the concepts of equity and numerical superiority/inferiority regarding number sense. However, developing number sense in young children does not seem to be an easy and straightforward task. For instance, children may be confused about the meaning of cardinal and ordinal numbers of toys in a set (Cooke, 2005). They might have difficulties in one-to-one correspondence such as counting up to seven objects in the right order but touching some of them more than once simultaneously (Charlesworth et al., 2003).

It is generally presumed that the adventure of mathematics was initiated by different groups of human beings with separating *one* and *many* to get the control of their environment for surviving and having more qualified life. (Boyer & Merzbach, 1991; Burton, 2006). Such needs made them to keep records of concrete objects in daily life. The records meant the concepts of number and equity, which was a common abstract property of the concrete objects, through one-to-one correspondence (Boyer & Merzbach, 1991; Cooke, 2005). The necessity for numbering plenty of the objects made the ancient humans to group these objects in various representations in order to easily count and recognize the product (Boyer & Merzbach, 1991). According to Burton (2006), using *tally marks* and *pebbles* were two of such artifacts around 30.000 B.C. The oldest tally marks were found in today's Czech Republic as 55 parallel line segments drawn on a wolf's bone in groups of five (Cooke, 2005). Pebbles meant small stones that were round and smooth in shape.

In 1874, Ernst Haeckel forwarded his theory of biogeny as "ontogenesis recapitulates phylogenesis". It was then adapted to psychology by Piaget, Garcia and Vygotsky which paved the way for association in the various fields of educational research. It was also employed for using history in mathematics education based on the main idea that "To really learn and master mathematics, one's mind must go through the same stages that mathematics has gone through during its evolution." (Jankvist, 2009, p. 239). In other words, there is an expectation for the re-emergence of certain obstacles and difficulties encountered in the evolution of mathematics while learning it. We could consult sources in history of mathematics (HoM) in order to prevent problems that might occur in understanding mathematical concepts (Sfard, 1994). In history, human ancestors initiated mathematics by manipulating concrete materials rather than a direct usage of abstract number symbols. There may be a parallelism between the ancient humans and young children of the modern day in respect of being mentally ready for learning the concept of number. Herein, we inspired from the notion of parallelism rather than completely aimed at advocating it. We thought that if young

children, who were at the beginning of their mathematical account, were instructed through early historical artifacts that used materials as alternative and simple kinds of representation while dealing with numbers, they might overcome the possible obstacles/difficulties hindering number sense acquisition. Moreover, HoM can be employed for keeping motivation and engagement high, revealing the human and cultural face of the subject (for details, see Tzanakis & Arcavi, 2000) through storytelling and pictures with young children. In order to investigate our claim that historical teaching modules might enhance young children's number sense achievement, we set the following research question:

- Does the historical teaching modules have potential effects for developing number sense in young children aged 5?

In order to include HoM in mathematics education, there are mainly three different ways: (1) *illumination approaches* in which historical factual information was presented as supplementary to the routine mathematics classes, (2) *modules approaches* requiring mathematics instruction through specific cases based on history, and (3) *history-based approaches* meant mathematics courses wholly built on historical perspectives (Jankvist, 2009). In this study, the *modules approaches* kind was adopted since we focused on the process of young children's number sense acquisition with the two historical-didactical materials within two class periods. It may also be characterized as Tzanakis and Arcavi's (2000) *historical package* in which a mathematical topic from the curriculum of interest is taught by means of historical-didactical materials in a relatively short period of time.

METHOD

Design of the Study

In this study, we aimed to make an in-depth analysis of the teaching and learning process in the historical teaching modules designed for developing number sense in young children aged 5 within a natural setting; hence, it could be characterized as case study (Creswell, 2009).

Participants

The study was conducted in a pre-school classroom in Ankara, Turkey. The participants were consisted of 12 children (five females and seven males) who were aged 5 years. The children would enrol in primary school in the next academic year.

The children had some background knowledge of numbers which was gained during the pre-school education. The children were expected to have the abovementioned knowledge and skills that were defined by MoNE (2012).

The Historical Teaching Modules

The two modules were based on different stories made up by the researchers in order to enable the children engage in the two historical artifacts including tally marks and

pebbles. The modules were implemented in classroom environment where children felt comfortable. One of the researchers showed pictures related to the stories and and guided the children while working on tasks. The other fluently read the stories and dramatized certain parts in them. Both of the stories included words and/or phrases such as *centuries ago, in very old times*, *times when the technology was not invented* to emphasize that the events occurred in the past times. As for the teacher, she was in the classroom as a non-participant observer during implementation of the modules. The modules were conducted in one class hour (45 minutes) within two consecutive weeks.

The first module was named as ‘‘Retired Island”. Black and white pictures about the story (referred to the old times), wooden sticks, papers and crayons (to draw tally marks) were used as concrete teaching materials. The children were gathered around a table to easily see the pictures and to record the amount of wooden sticks on their papers. Before getting started, transition questions such as “Have you ever gone to the sea?” and “Have you ever seen an island?” were asked to prepare the children to the content of the story. After taking the answers, a world map was showed to tell where the story happened. Some other pictures of a ship, storm in sea, a retired island, Captain Jack, a bag on beach, woods, and a boat were showed in parallel with the story. After the story was read, it was explained that the children should help Captain Jack in order to count the number of woods for constructing his boat to escape the island. Additionally, it was stated that the captain determined to record this work in his diary. The children were asked for how to count the woods (seven wooden sticks). They were invited to draw tally marks on their paper for recording the number of wooden sticks in the meantime. After this task, it was announced that the boat sunk due to the lack of the woods. The number of wooden sticks was increased to 10. A discussion created about how to notice the higher number of woods in the record this time, and the children were guided to feel the need of grouping the tally marks.

“A Shepherd in Kuka Tribe” was the name of the second module. The materials employed in this module were black and white pictures about the story (referred to the old times), sheep pictures and pebbles. The children sit in a half circle position on the floor to provide interactivity. At the beginning, the children were asked some transition questions such as “Have you ever seen a forest?” After sharing their experiences, a world map was put on the floor to show where the story occurred at this time. During the story, some other related pictures as rain forest, a mountain, sheep, a house and a shepherd were also shown. When the story finished, the children were invited to help the shepherd so as to count and record his sheep. They were initially asked for how to count and shared their ideas. After that, nine sheep pictures and pebbles were separately placed on the floor. The children were expected to count and record the number of sheep on their own ideas. They also invited to group the sheep and corresponding pebbles in different ways.

Finally, the children were asked some open-ended questions related to the objectives of the modules.

Data Collection and Analysis

In order to collect the data, the ethical approval document was provided from Research Center for Applied Ethics in Middle East Technical University in Ankara. The pre-school administration gave permission for conducting the study with one of the groups of five years of age after being informed about the ethical considerations.

Data were collected through anecdotal records of the implementation of the two modules and semi-structured interview with the early childhood teacher at the fall semester of 2012-2013 academic year. Anecdotal notes were taken for the children's observed behaviour and speech during the implementation.

Data analysis was made considering Dunphy's (2006a) four aspects for young children's number sense, which she introduced in her comprehensive dissertation. She set a framework number sense in young children who were around five years of age. The framework has four aspects as *pleasure and interest in number*, *understandings of the purposes of number*, *ability to think quantitatively*, and *awareness/understanding of numerals* (2006a). The children's pleasure and interest in number is their disposition towards practices regarding numbers such as being motivated for games that can be played with numbers. Understandings of the purposes of number refers to having some idea about the reason behind humans' need for numbers, for instance, being interested in why numerals written on the home telephone. As for the ability to think quantitatively, it addresses "to count, subitize, and estimate, and to relate numbers to each other" (Dunphy, 2006b, p. 58) which can be exemplified with proficiently utilizing concrete models for counting. Lastly, awareness/understanding of numerals denotes knowing the language of numbers (i.e., symbols corresponding numbers). For example, the children should notice the numerals written in an elevator in order to go home upstairs. This framework can be employed for both describing young children's existing number sense and revealing their relevant learning in progress (Dunphy, 2006b).

FINDINGS

Pleasure and interest in number

It was remarkable that the children were quite interested in the historical modules. Both of the stories supported by black and white pictures took their attention and kept them silent. It can be asserted that the storytelling made them mentally ready for the actual tasks. The stories, which gained inspiration from the needs for counting and keeping records in the history, made them engage in the tasks. Additionally, the concrete materials as pictures, tally marks, wooden sticks, pebbles, and sheep cards also aroused an interest on the grounds that they visualized the number related tasks.

In the first module, the children loudly counted the wooden sticks one by one and drew the corresponding tally marks simultaneously. It is possible to state that being actively involved in drawing tally marks by a hands-on experience kept their interest alive in

performing the task. At the end, the children were asked for what they liked or disliked in the module. It was observed that the children were impatient to answer the questions. 11 children noted that they liked the module and underlined the most liked activities *drawing tally marks*, *seeing pictures*, *counting*, and *saving Captain Jack*. One of them expressed that she did not like since they studied on the desk and added that she liked the *play* in this module.

As for the second module, saying that we had another story for them made the children be silent. One of them explicitly stated his wonder. Such behaviours might be a result of their positive experience in the first module. They were also concerned with pebbles, which were not used in the previous activities, and seemed to be impatient for using them. After the story ended, there was a chaos among the children for doing the task with sheep and pebbles. This time all of the children liked the module and highlighted the following activities and elements: *using stones*, *seeing pictures*, *sheep*, *story*, *putting stones for each sheep*, *asking questions*.

Understandings related to the purposes of number

In both of the modules, we aimed to contribute the children's understanding related to the purposes of number through stressing that human beings needed for numbers and that everyone could do/study mathematics. Related discussions could be generated with open-ended questions (Q) that displayed the modules' potential for achieving these aims. The following dialogue from the first module illustrated such discussions with the children ([A] represented the answers of the children):

Q: Is mathematics necessary?

A: Yeees!

Q: Why do we use mathematics?

A: Humans build ships.

Q: Who do/use mathematics?

A: (1) Students. (2) Workmen.

In the second module, we had the related dialogue below:

Q: What should the shepherd do in order to find the number of the sheep?

A: [He] should count.

Q: So why do we use pebbles for this?

A: (1) [They] enable us to know it is two. (2) One pebble is put for one sheep.

Q: Why it is one?

A: (1) In order not to count the same sheep again. (2) In order to prevent being confused.

Ability to think quantitatively

It was aimed to contribute the children's perceptions about one-to-one correspondence, equity, cardinality, ordering and grouping as abstract properties of objects through using of concrete materials in the two modules.

The children proficiently matched seven wooden sticks one by one with tally marks, which they drew on their own, in the first module. They also counted up to seven in the right order at the same time. When the wooden sticks increased 10 in number, the children were guided to group and asked for the reason behind grouping. They presented the relevant arguments such as "To make them five." and "To separate them." They finally decided to group in three and consider the last tally mark as the rest.

The second module seemed to be more meaningful for children in respect of the abstract properties in question. Here three related dialogues were demonstrated:

Q: Why did you make in this way (by showing nine sheep matched with nine pebbles)?

A: (1) They will be equal. (2) One pebble is put for one sheep.

Q: Why?

A: There will be one stone on each of the sheep.

…

Q: How did we count?

A: In an order with pebbles.

…

Q: What should the shepherd do if he has many sheep like this (by showing nine sheep)?

A: (1) If he has many, he may bring the half to his house. (2) Shall we divide? (3) By grouping.

Q: So how can we group?

A: (1) We drew a line at the centre (in the previous class). (2) We may group in five?

After noticing that there was an asymmetry with putting five pebbles in an order and four pebbles under that order, they changed their mind as: "In three!"

A: (1) In three! We separate sheep in three and also pebbles in three. (2) Three by three finally makes nine, then we put the pebbles as nine. It is equal!

Awareness/understanding of numerals

The modules did not seem to influence the children's awareness/understanding of numerals since the historical modules did aim to study mathematics verbally rather than to enhance his recognition with abstract number symbols.

DISCUSSION AND IMPLICATIONS

This study indicated that teaching modules in HoM have potential to assist the development of young children's number sense under the three aspects identified by Dunphy (2006a) as pleasure and interest in number, understandings about the purposes of the numerals, and quantitative thinking skills. Regarding the first aspect, telling stories linked with the two historical artifacts, showing pictures inspired by the old times and using relevant concrete materials seemed to make their attention alive. Smestad and Clark (2012) also pointed out the contributions of using HoM with younger children to launch their interests and mathematical thinking through storytelling and multiple representations. As for the second aspect, tendency of the children in explaining the need for mathematics in daily life might because of the content of the two stories including instances about the use of mathematics. In the third aspect, physical materials in the historical artifacts might make the children's learning more meaningful as the related discussions indicated. Children at early ages need to learn by concrete and hands-on experiences (Hohmann & Weikart, 1995). Moreover, providing opportunities for children to reflect their thinking with concrete materials was suggested on the grounds that it could significantly contribute to quantitative thinking and understanding (NCTM, 1989). Drawing tally marks and using pebbles might strengthen the children's ability to think quantitatively from the aspect of making the abstraction process more visual (Carruthers & Worthington, 2005).

The teaching modules could be characterized as usable in terms of the concrete materials used and the time spent. Conducting the modules required materials which were easy to find and cheap. The modules also took reasonable time in terms of implementation which was an issue of concern in the field of using HoM in mathematics education (Tzanakis & Arcavi, 2000).

According to Carraher and Schliemann (2007), early years are critique and important for young children to gain number sense due to the fact that it took long time. In this context, early childhood teachers as the implementers of early years mathematics should also be trained on how to use HoM as an alternative way for developing similar mathematical understandings in young children. Further research should be made on how HoM could be an inspiration for developing similar teaching practices.

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