THE HISTORY OF 5TH POSTULATE: LINKING MATHEMATICS WITH OTHER DISCIPLINES THROUGH DRAMA TECHNIQUES

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We present the design and implementation of a cross-curriculum project concerning the history of 5th postulate that was carried out among 11th grade students in a public school in Athens. We used History of mathematics as a unifying framework for an interdisciplinary curriculum through 'Drama in Education' techniques. Drama in education contributed to a creative way for the students to reflect on broader learning issues (emotional, cognitive, social and political) mostly because of Drama experiential dimension. Students' active involvement through this framework helped the students to conceive both mathematics and other disciplines knowledge in a meaningful context and made them realize that mathematics is a cultural and social construction; a component of our world.

INTRODUCTION: HISTORY OF MATHEMATICS AND INTERDISCIPLINARITY

Specialization in education is a modern phenomenon, which results in viewing present day school mathematics as completely separate from other subjects of the curriculum. Meanwhile, school administration and timetabling of classes also often work against efforts to make links between subjects (Grugnetti and Rogers, 2000: pp.52) making mathematics teaching ineffective (Anice, 2005).

This finding leads to the reassessment of the content and way of teaching mathematics. Interdisciplinarity is suggested as a way of "injecting" meaning to mathematics. This rationalization process is associated with personal experiences and the collective engagement in an activity. Interdisciplinarity is perceived as a complex process which, in the context of teaching design, requires the detection of each child's personal rational experience and the cooperation in groups based on common interests in complex matters (Kalavasis, 2008: pp. 24).

Interdisciplinary methods strive to create connections between traditionally discrete disciplines such as mathematics, the sciences, social studies or history, and English language arts (Coffey, 2011). During interdisciplinary teaching, educators apply methods and the language of more than one academic discipline to examine a theme, issue, question, problem, topic, or experience, enabling students to see the links between various subjects and consequently to see the knowledge in a unified way, making their school experience more compatible with their real life. The objective of interdisciplinary teaching can be orientated towards content, method, competence and ways of thinking,

as well as to the characteristic forms of subject-, parallel- and comprehensive-orientation (Bechmann, 2009:pp.32).

Throughout the literature we encounter different kinds of interdisciplinarity. In Jantsch's (1972) five level taxonomy of disciplinarity the three middle ones that may be considered as 'true' types of interdisciplinarity are: *pluridisciplinarity,* when various disciplines, usually at the same hierarchical level, cooperate without coordination about a common theme, *crossdisciplinarity,* when the axiomatics of one discipline are imposed for support upon other disciplines *and interdisciplinarity (proper)* when there is coordination in all involved disciplines by a higher level concept (Jankvist, 2011).

Research has shown that interdisciplinary and cross-curricular teaching can increase students' motivation for learning as well as their level of engagement and thus provide conditions for effective learning (Thaiss, 1986). Barton and Smith (2000) explain that interdisciplinary units enable teachers to use classroom time more efficiently and address content in depth, while giving students the opportunity to see the relationship between content areas and engage in authentic tasks. '*Interdisciplinarity is increasingly viewed as a necessary ingredient in the training of future oriented 21st century disciplines that rely on both analytic and synthetic abilities across disciplines'* (Shriraman, 2009: v).

The History of mathematics provides a suitable framework both for integrating varied disciplinary areas of curriculum, as well as for helping students with their understanding both of mathematics and other subjects. Keeping in mind that the history of mathematics as the history of ideas is strictly linked to the history of human beings, we have to analyse the cultural, political, social, economic contexts in which ideas arose (Grugneti and Rogers, 2000). Interdisciplinarity through history of mathematics can reveal this wider aspect of mathematics as a cultural activity; as a human activity both done within individual cultures and also standing outside any particular one.

This integration of history is not confined to traditional teaching delivery methods, but can often be achieved through a variety of media such as doing projects, watching films, constructing models, researching history in libraries, devising dramatic presentations, surfing the www, which add to the resources available for learner and teacher (Nagaoka, 2000).

In this paper, based on our research results, we claim that the use of 'Drama in Education' in Mathematics teaching motivate students to reflect on broader learning issues (emotional, cognitive, social, and political) mostly because of Drama experiential dimension.

HISTORY OF MATHEMATICS AND 'DRAMA IN EDUCATION'

Drama in Education is (DIE) is a form of theatrical art in which the child while creating and playing roles, projects himself into fictional characters and situations, exploring and expressing his ideas with his body and his voice (Alkistis 2000: pp.78). It is a highly structured pedagogical procedure utilizing specific rituals and techniques of dramatic art which grants us, through the creation of an imaginary world, the context for teaching a notion, an idea, a fact, a solution of a problem as well as the potential of cultivating personal and social skills. It constitutes a dynamic and creative tool for the teaching of different subjects of the syllabus through collective and experiential activities (Andersen, 2004), putting children in the position of the actor (experience), audience (critical ability), author (meaning) and director (form) (Neelands 2008). DIE combines a) form and content b) action and reflection c) logic and imagination d) thought and emotion and e) body and soul.

Regarding the teaching and learning of mathematics, the research of the influence of introducing drama in the teaching process, limited in primary and junior high school, is very encouraging in students' understanding and retention of mathematical notions (Saab, 1987, Omniewski, 1999, Fleming et al., 2004, Duatepe, 2004) and in creating positive impact in their attitudes towards mathematics (Duatepe, 2004). Specific techniques such as the 'as-if' approach can create the context for teaching a concept, an idea or an event and offer opportunities for exploring mathematics in a variety of historical, social, political and cultural contexts (Kotarinou et al., 2010, Ponza 2000b, Pennington and Faux, 1999).

THE RESEARCH

Pragmatological data for this paper arose from our research on exploring the dynamics of Drama in Education Techniques in teaching Geometry in high upper school. This paper discusses the following research question: does and in which way interdisciplinarity through history of mathematics in a DIE setting affect students' learning as well as their epistemological beliefs about mathematics?

The setting: the research was carried out in a group of 26 (11^{th}-grade students) from different directions of studies in the 2^{nd} High School of Ilion (Athens, 2010-11) during four months.

The method: we designed and implemented an interdisciplinary didactical intervention, a didactical experiment (Chronaki, 2008). The didactical experiment concerned the teaching, through Drama in Education techniques, the axiomatization of Euclidean and Non-Euclidean Geometries as well as the history of Euclides' fifth postulate. The question of judging the effectiveness of integrating historical resources into mathematics teaching may not be susceptible to the research techniques of the quantitative experimental scientist. It is better handled through qualitative research paradigms such as those developed by anthropologists (Fauvel & van Maanen, 2000, pp. xvii).

Exploitation of ethnographic research techniques (observation-interviews) helped us gathering research data, while some discussions in the classroom were audiotaped and all students' presentations were videotaped and analyzed.

The interdisciplinary project 'The history of 5th postulate. From Euclid till non-Euclidean geometries': Within this paper we present the unit of the teaching experiment which concerned the history of the fifth postulate as one of the five set by Euclid and it being challenged as an independent one by mathematicians of classical Islam and by the West mathematicians of the 19th century, untill its replacement following its refusal by Bolyai, Lobatscevski and Riemann.

The researcher in teaching role carried out the project in 18 teaching periods, in Geometry, History, Literature, Greek Language, and Ancient Greek Language class. The three main points of the interdisciplinary project were: the mathematics per se, the Mathematicians and the historical, cultural political, social, economic contexts in which ideas arose. Our teaching aims were: a) the students acquainting with the axiomatization of Euclidean Geometry b) the students perceiving the role of the postulates in the axiomatic foundation of a science c) through the errors of different proofs of the fifth postulate, the students to acquaint with the various equivalent with the 5th request propositions that characterize Euclidean Geometry d) students perceiving that Mathematics is constructed through its development located in various specific historical, geographical, cultural and social contexts. The project consisted of the following units-steps:

1. *Euclid's Elements and the axiomatization of Euclidean Geometry* (6h): The relation between Euclid's axiomatic foundation of Geometry and Aristotle's 'Logic' was studied as well as the definitions, the Postulates and Common Notions from the Book I of Euclid's 'Elements'. Knowing from Thomaidis and Tzanakis, (2010) research, that the study of original texts in the classroom creates a new didactical environment in which students actively participate in the classroom discourse and exhibit a positive attitude towards the subject, which never happens in conventional geometry teaching, we gave students, in Ancient Greek language class, excerpts from Euclid's *s* original texts and requested them to read and translate the text. In Geometry class, students in groups were asked to answer questions concerning the mathematics of the text, after having studied some relevant excerpts of the book 'The Historical Roots of Elementary Mathematics' by Bunt Jones and Bedient (1976). Then students prepared and had performaces concerning the postulates, the common notions and some definitons of Euclid axiomatic foundation of geometry, using drama techniques as 'role-playing', 'reportage', 'alter-ego', 'interview'. We must notice here that in this unit students for the first time came into contact with Euclid's own form of the fifth postulate.

2. *Euclid and the historical, cultural and political frame of his era* (4h): The purpose of including elements of the history of mathematics has to do with showing the students

that mathematics is dependent on time and space, culture and society, that mathematics is not ‘God given’ and that humans play an essential role in the development of it (Jankvist, 2009:2734)**.** For this reason, in order the students to know the historical context in which Euclids’ Elements were written, a digital presentation was held by the researcher in History class, concerning Alexandria in the Hellenistic period, while excerpts from the book "The Parrot's Theorem" were read concerning the history of this era, as well as the reasons for the blossoming of Mathematics in this historic period and area. Knowing that Dramatization is an important tool in the repertoire of a teacher for humanising and contextualising the development of mathematical concepts (Hitchcock 1996a; 1996b; Ponza, 2000a, 200b), the chapter ‘Euclid’s conceit’ from J. P. Luminet book ‘Euclid’s bar’, presenting the mathematician Euclid and his era, was read expressively by some students in Literature class, while some scenes of the same chapter concerning differences in thought between Pythagorians and Euclid, as well as historical anecdotes about Euclid were dramatized.

3*.‘History in shadow’: the controversy of Euclid’s Fifth postulate till 18th century (5h)*: There is a common belief held by many, teachers and students alike, about the static nature of mathematical concepts (Jahnke, 2000). The history of Euclid’s Fifth postulate provides the potential to undermine this entrenched perception. In mathematics class the unsuccessful efforts of Arabs mathematicians Thabit ibn Qurrah, Al-Haytham, Omar Khayyam and Nasir al-Din al-Tusi as well as Saccheri and Lambert to prove the famous Euclid’s fifth postulate were presented while equivalent to the 5th postulate propositions were presented to interpret the mistakes made in these efforts. In history class the development and the reasons of development of Mathematics in the Islamic world, from the 8th to the 13th century AD, were discussed. For this reason, a short extract from the book *‘‘The Parrot's Theorem’’* of Denis Guedj (pp. 269-272), which refers to the ‘‘House of Wisdom’’ in Baghdad and its role in the collection and translation of the work of the ancient Greeks, was read. A combination of ‘Shadow Theatre’ and ‘role playing’ was used for presenting all these unsuccessful efforts as well as the mistakes made at the proofs.

4. *János Bolyai, Lobatscevski, Riemann, the founders-creators of non Euclidean geometries (3h*): As the biographical allusions serve the purpose of humanizing concepts (Ponza, 2000a) students, in Greek language class, studied the biographies of the two latter mathematicians from Bell’s book ‘*Men of Mathematic's’’* and quotes from letters from Gauss to F. Bolyai and berween father and son Bolyai.

Integrating history of mathematics invites us to place the development of mathematics in the scientific and technological context of a particular time and in the history of ideas and societies (Jahnke, 2000). Hence, we emphasized that many times dominant philosophical stances during a historical era act as obstacles to the challenge and overcome of assumptions in mathematics, as well as to the publication and widespread

acceptance of new mathematical theories. An example lies in our case of concern, the controversy of the uniqueness of Euclidean Geometry. Within this context we mentioned the example of Immanuel Kant, the dominant philosophical physiognomy of the century that preceded the discovery of non-Euclidean geometry, whose ideas about space and geometry acted as an obstacle on the acceptance of non-Euclidean geometry. Some attribute the non-publication of Gauss's ideas on non-Euclidean geometry just in his unwillingness to conflict with Kant (Davis, 2007: pp. 133). In this unit different Drama techniques as 'Letters', 'Role on the wall', 'Conscience alley', 'Conflicting advice' were used for presenting their work and some snaps shots of their lives.

RESULTS AND DISCUSSION

Analysis of the dialogues in students' Drama performances suggests that students conceived the mathematical notions that they had been assigned to present, integrating them correctly in their dialogues. As Ponza (2000a) refers: '*This method of teaching is not just intuitive. When students write or dance or perform mathematics they work out, they analyze, organize and solve'.*

The students' answers in the interviews, two months after the end of teaching experiment, indicate that students themselves believed that through Drama based instruction they learned better and easier.

-(A)The way we presented, using theater, it isn't that we just sat up and said it, we had to prepare it, and this required to have understood it.

-(I) in the way we did all these, I think that one learns more easily. What we learned (through this procedure) wasn't designed for learning by heart or to show to the teacher that we really learned that. I think it was easier to learn in this way and to retain this knowledge in our mind.

- (G.) Well. Until now, geometry was completely indifferent to me, and still is, if they teach it the way they do. Because to learn something by heart it is not nice, while if there is a story it is a little more interesting, while if you do a sketch it is nicer and it becomes a more interesting lesson. Because you learn more this way.

What was the main effect of Drama in students' learning? In this teaching experiment, due to students' responses, it was Drama that motivated all students' active participation in the teaching experiment.

-(S.) If we have done all the tasks except the presentations, we would have been bored to death.

- (N.) Without drama, I don't think that anyone would be interested (in the project).

-(V.) It was completely different than just writing stuff, from comprehending something yourself and trying to pass it to others. It was very nice.

How did interdicsiplinarity affect students?

• ***In unifying knowledge:*** Interdisciplinarity enabled students to see the links between various subjects and consequently to see the knowledge in a unified way.

-(St.) It was something very different, something not expected and we had to wait till this grade of high school to be able to do something similar, a similar activity that is related with something different from the syllabus. Indeed the specific issue that connects Mathematics with different disciplines as Ancient Greek, History, was very interesting. It was interesting because we talked about Euclid but not only about the postulates and the various theorems, but we talked about the historical and cultural context that prevailed in Alexandria at that time..
-(J.) This was interesting and we had not done something like this before. I think this did History, which I do not like, more interesting because I learned about mathematicians and I like this more.

• ***In students' beliefs about Mathematics' historical and cultural context:*** Students experienced a diachronic evolution of geometry and its interaction with the historical, the scientific, and the cultural evolution, as well as with social, economic and political conditions of each era. The introduction of mathematics history into the course curriculum attracted the attention of pupils, since it was the first time that they had such an experience. Students welcomed the activities related to the history of geometry, which linked the science back to the specific historical, political and economic context within it developed

-(Char.) It was nice that we learned about this era, while in the ordinary lesson we didn't learn anything about this, it was just triangles, squares.
- (P.) In general it is nice to observe in every science its history. To connect every science and every discovery of science with the historical events of that time, in order to see the mentality of these people and the way they functioned and reacted and to see the stimuli they had at that time.

From students interviews it appeared that there were students who linked the development of mathematics with the economic development of the region in which it was developed.

-(St.)We realised that in order for the science to be evolved, there needs to be evolution in all other areas first, starting with financial areas and then cultural ones'.

• ***In students' beliefs about Mathematics as a human creation:*** From the follow-up answers in the interviews concerning students' beliefs about the evolution and development of mathematics, it seems that students believe that mathematics is timeless and superhuman and ahistorical.

-(A) The historical connection of mathematics is very important. Mathematics seemed like having come from heaven. We had just been told the 1 and 2 and the numbers and the calculations, all ended there. The historical connection of mathematics is very important, as it is its historical evolution as well as that of Geometry.

- *(V.) Previously, I thought that the knowledge of mathematics came from God, from heaven. I couldn't be in the process of thinking that someone thought of it. Basically I was thinking of that, but I was wondering: how can one think of it? What has he got in order to be able to think of it?*

Through the efforts of proving Euclid's fifth postulate and through the mathematicians' biographies, students perceived mathematics as a historically evolving human creation and also realized that other cultures also, have contributed to what is defined as Western mathematics.

- *(Tz.) It was nice, because if you only see the result it is boring. You must learn a little bit of what has happened in the past; who created it, so it was nicer, I learned more things.*
- *(Th.)...We saw how theorems and postulates have been created and it was interesting, because it is difficult to understand them if you see them in their final form.*

Students met for the first time the struggle of many mathematicians that has not been crowned with success. They emphasized the continuous and strenuous effort that was made by the mathematicians of the wider Islamic world and later by Renaissance mathematicians, to prove the fifth postulate.

-*(Tz) In the beginning I said, well, as they don't find it, why don't they stop? Yet they have been trying and trying. They liked all these too much.*

We asked students to tell us their thoughts from their acquaintance with the biographies of Bolyai, Lompatsevski. and Riemann. Students in their responses highlight the dedication of all these people in mathematics and the perseverance in their objectives.

- *(Mar.) ...I saw that these people in a great part of their lives were dealing with mathematics and they had set a target, trying to reach it. It was very interesting, all this effort to prove something, really interesting.*

CONCLUSIONS

Our research offers considerable evidence of the effectiveness of History of Mathematics in linking mathematics with other school disciplines. History of Mathematics became an open door towards interdisciplinary work and consequently towards a wide range of possibilities for all subjects. Students had the opportunity in language class to write texts, which included mathematical discourse, while in History class for the first time they experienced mathematics as a cultural treat. This indisciplinary approach was the key role for the historical and social context in which mathematical concepts were created to be illustrated and provided students the opportunity to modify their epistemological conceptions about mathematics. Likewise, cultural construction and the contribution of different cultures in the creation of mathematics were showcased. Drama provided students the incentive to work while through drama techniques students were able to experience all these dimensions of mathematics, not only mentally but emotionally and physically.

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