KINDERGARTEN CHILDREN'S REASONING ABOUT BASIC GEOMETRIC SHAPES

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In our study we attempted to investigate the criteria used by preschool children in distinguishing basic geometric shapes, namely circles, rhombus, squares, triangles and rectangles. Besides the detection of the syncretic level as it was described by Clements et al. (1999), analysis of our observations revealed seeds of relational thinking in very early age, which via suitable collaborative tasks designed by teachers may advance preschoolers geometric thinking.

Keywords: geometric shapes, categorization, syncretic level, relational thinking

INTRODUCTION

Researchers' interest in preschool mathematics has considerably grown during the last ten years. Many research papers have been published and lately a book by Levenson, Tirosh and Tsamir (2011) entitled Preschool Geometry: Theory, Research and Practical Perspectives has appeared, while during the 6th Conference of European Research in Mathematics Education (2009) a new working group in Early Years Mathematics was established. Yet, geometry and spatial thinking are often ignored or minimized in early education (Sarama & Clements, 2009).

During preschool education children's concept images are based mainly on perceptual similarities of objects in their immediate surroundings and personal experiences. Gradually, through instruction, children pay particular attention to certain common attributes of shapes, and begin to develop informal definitions of shapes; they usually "define" a shape detailing his properties. Understanding of geometric concepts involves an invocation of a set of mental images along with a corresponding set of properties for that class of objects (Fischbein, 1993). Vinner and Dreyfus (1989) pointed out that students might not understand a concept in depth, if they do not bond the "concept image" and "concept definition" appropriately.

In the research, students' understandings of geometric figures are mainly analyzed taking as framework the van Hiele's levels of geometric thinking (van Hiele, 1986) and the hypothesized geometrical shape learning trajectory developed by Clements and Sarama (2009). Clements and Battista (1992) enriched van Hiele's theory, suggesting a sixth level of thinking more primitive to the "visual level". At this *pre-recognition* level children perceive geometric shapes by attending to only a part of the shape's characteristics: "[…](*Children) may distinguish between figures that are curvilinear and those that are rectilinear but not among figures in the same class; that is, they may differentiate between a square and a circle, but not between a* *square and a triangle.*" (Clements & Battista, 1992, p. 429). Gradually, experiencing multiple examples of a shape, children mentally construct increasingly elaborated shape-schemas, which engender the formation of mental prototypes (ibid., p. 208). If children's schemes remain chained to a prototype this prototype may be the source of incorrect responses, especially in the transition phase to the next description-level. (Clements & Sarama, 2007, p. 505). Furthermore, Clements and Battista suggested that geometric levels of thinking coexist. They used the term 'syncretic level' in order to indicate *"…a combination of verbal and imagistic knowledge without an analysis of the specific components and properties of figures. At the syncretic level, children more easily use declarative knowledge to explain why a particular figure is not a member of a class*" (Clements & Battista, 1992, p. 211). Recent early childhood theorists have challenged the developmental approach and the academic content approach as, from a historical, and sociocultural perspective, both positions limit what children could learn (Kessler & Swadener, 1992; Lubeck, 1998).

Our research tries to shed new light on the ways pre-school children construct geometric concepts through an investigational context that promotes communication and language exchanges. Basic assumption of this work was that a structured teaching setting with emphasis on language exchanges between participants, on the design of meaningful for children activities and on literacy practices, can promote the construction of school knowledge categories such as geometric shapes as well as children's reasoning development. In fact, the data of this paper derive from a more extensive linguistically oriented research grounded on the sociosemiotic perspective of Systemic Functional Linguistics[1] (SFL) and its model of context analysis, which interrelates the socially constructed knowledge (semiotic representation of reality) with the realization of language in context-situation (Giannisi & Kondyli, 2011; 2012a; 2012b). The results here, concise the basic conclusions concerning the identification and classification of all basic shapes used in the research, as these are elaborated in order to address to a more specialized public. From this point of view, our venture gets the meaning of a cross examining revision.

DESCRIPTION AND METHODOLOGY OF RESEARCH

A total of 18 students from a sub-urban public kindergarten school participated in this study. The older students of the class (12 girls and 6 boys, aged 5:6 – 6:3) worked together with their teacher-researcher in small groups of 3 (2 girls and 1 boy) during the period between April and May of the school year 2008-2009. The corpus comprises of 20 video and tape-recorded meetings (about 13 hours in total).

Children's involvement in geometric concepts was set within a scenario frame of action: the production of books, which were to be sent to schools that couldn't afford to make such a purchase (action proposed to students on the occasion of the International Children's Book Day). Each group undertook the task of creating a book about geometric shapes as well as an insert card game of geometric shapes. The book to be produced was described from the beginning as a "learning book" (i.e. an informative leaflet/dictionary of "basic terms") for children. In this case, the knowledge of geometric shapes was considered to be a precondition for the receivers to play the card game found in the book. The whole project was carried out during the successive meetings of each group, and was divided into activity modules, coherent and consistent with each other, according to the original context of the project (the production of the book), and, partly, to the level of their difficulty. The general objectives of each meeting's tasks were clearly explained at the beginning of the session. The teacher participated in the discussion as a coordinator, asking open, general questions, related to the aim of each activity. Special attention was given so that the teacher would not direct children's answers but encourage the discussion between the members of each group.

In brief, the tasks of all groups included the following activities:

A] Classification of flat geometric shapes (circle, ellipse, triangle, rhombus, square, rectangle) which would be pasted on the pages of the book after discussion and agreement between the members of the group (36 non identical paper geometric shapes of the same color and texture, with variations within each category regarding the shape qualities and size of the shape).

B] Addition of texts: Giving "titles" to every shape category (*categorization by naming*) as well as producing informative texts for the reader (*definitions*, *descriptions of their categorizations*), which were dictated to the teacher.

C] Creation of book's card game (a deck of 27 cards of 4 basic shapes: circle, triangle, square, parallelogram) -after the completion of phase B- and a sample performance of the game in a whole class meeting according to instructions read by the teacher. In order to design the game, each group was asked to collect objects found in the classroom and to classify them in different baskets by shape, according to the selected facet (designed game would preserve only the shape categories and the total number of cards of the initial game). Children were given samples of empty cards and asked to find different objects (at least 6) fitting to the size of the card in order to draw (outlining object) the 4 geometric shapes. Suitability and classification of the objects were then discussed within the group and each child undertook the drawing of 8 cards (picking 2 objects from each basket-shape category).

D] Presentation of all groups' books with the participation of the students of another kindergarten class and performance of a description game (a child picks a card from the deck and describes the shape depicted on it to the rest of the students, who try to identify it).

The whole project presupposed a) collaboration so that a common target (the production of the book) could be achieved b) linguistic interaction and negotiation of meaning in order to agree and compose book's content.

For the purposes of the research, each set of activities could function as a complementary source of information and as a means of checking coherence and progress of children's response.

RESULTS

Two sets of data are analyzed in this paper. The first set of data includes children's attempts to classify and define shapes according to the demands of the interrelated/ closely connected activities of Task A and B. A second set of data includes the subsequent descriptions of shapes during the 'description game' (task D). The results presented below concerning identification and classifications of shapes focus on the decisions taken by groups of children. The accompanying excerpts of dialogues, although necessarily restricted here, are indicative of the procedures across tasks' performance, as well as of the semantic choices and of classification criteria constructed within groups.

Circles-ellipses

Based on data from the collected material, both circles and ellipses were treated as closely related categories by all groups of our sample. During their final categorization, 3 out of 6 groups classified the items of the specific categories in a unified category under the name "circle". Moreover, in all groups there are statements which correlate circles and ellipses as semantic categories of a lower semantic organization level, within a general category called "round" or "cycline/ circular".

At the following excerpts, the group of children (G1, G2 etc) and each task of the project (A, B, C, D) are included in a parenthesis before the enumeration of each excerpt (e.g. G1A/). In dialogues the children are referred to with their initials and the teacher with the abbreviation (Tch).

(G1A/2) T.: *Madam, I gather all the round ones* (ellipses and circles).
(G3B/68) K: *And if we cut them like this (a virtual cut on an ellipsis), they become curves like circles*.
(G4A/84) Z: *This is the 'cycline' group* (shows only the circles).

However, in the discussion that followed about the items of the 'cycline' group, Z and A support the common inclusion of circles and ellipses in a group named "circles" (G4A/86), while N disagrees, supporting that ellipses are not circles. In that discussion Z. refers to a "circular group" including ellipses as well.

(G4A/86)
Tch: *Which ones are the circles?*
Z: *These and these* (circles and ellipses).
Tch: *Take a look you too N and A. I want you to participate. Are these all circles?*
Z: *This is the circular group, family (...) All these are circles, I think.*

Although circles and ellipses seem to be closely connected categories clearly distinguished by the others in task A, in the cases of groups of children who decided to make separate categories the distinction is accurate in 2 groups and almost accurate in one group, which added one ambiguous (circular) ellipse in cycles. The distinction between circles and ellipses is achieved mainly by reference to the circle prototype. Thus, children characterized ellipse as a "(a bit) stretched" circle, or "like a circle", "so and so", "(circles that) look like eggs".

(G1A/3)

F: *Well, normally* (indirect comparison to the circle prototype), *this one,* (showing an ellipse) *is a little more stretched, it's an egg.*

(G1A/4)

F: *It's a circle, but it's a bit stretched.*

(G1B/6)

Tch: *Yes, and you said that they are circles. Are they circles?*

F: *Like circles.*

K: *Like circles, but "so and so" she said.*

(G4A/86)

Z: *This is the circular group, family.*

Tch: *N, take a look. Do you agree that all these are circles? It's you that will decide.*

Z: *I think that they are all circles.*

N: *These are not circles* (he points at ellipses). *Those are* (he points at circles).

Z: *They are. But they look like an egg, that's why.*

It is noteworthy that the characterization "stretched"/ "slanting", as a critical distinctive quality between circles and ellipses, is also used with small variations in the correlations of square-rhombus, as well as for triangles in relation to the inclination of their sides ((G3A/75) K: *The triangles and some "slanting triangles"*). In all cases, children have a prototype in mind: circle, acute isosceles triangle. Besides, representations of such prototypes existed as a classroom shape chart. The wall-pictures function in an assistant way as a means of cross-examination, though children seem to use them also to enforce their arguments.

Triangles

In general the "triangle" appears to constitute a generally "obvious" and recognizable category of shapes, with which children are familiarized. Thus, there was no overlapping with the rest of the categories in the material, with the exception in some cases with the rhombus. The rhombus constitutes the shape most commonly associated with the triangle for reasons more thoroughly examined later.

In addition, some triangles are treated as more typical or representative, and some others appear to move along the edges, constituting a subject of discussion or disagreement within the groups, arousing the production of argumentation, which in turn brings into light the more typical qualities of the category (G3A/74).

In some cases, typicality does not stem only from comparisons between members of the category, but also as a function of their relative spatial placement, as implied by the usage of the word "upside-down" or "crooked/ stretched triangle" in the following excerpts.

(G2A/191)

G: *Oh, I stuck it upside-down!*

(G3B/210)

M: *I mean... Let's say this is a pair of scissors* (she holds a pencil) *and I cut it a little like this* (she points to a diagonal on a square). *How will it become?*

Tch: *What will it become?*

K: *A crooked/bent/stretched triangle.*

Concerning the hypothesis of the prototypicality of the shapes, in relation to their spatial arrangement, more research is needed. Nevertheless, from children's actions on the material, as well as from their corresponding comments and discussions arises a clear ranking at the representativeness of the members of the category "triangles". The shapes that children initially recognize as triangles are the two acute isosceles triangles and the acute scalene triangle (e.g. G3A/74). It appears that the main criterion for characterizing a shape as a triangle is the existence of three acute angles.

(G3A/74)

[Takes in hand first the 2 acute isosceles triangles and then the right-angled isosceles. Then grabbing the right-angled scalene triangle, asks]

M: *Should we make this a triangle too?*

Five out of six groups agreed on a common classification of all triangles in task A (except of a case G2A/41, where a child refuses to see the right-angled triangle as a "whole" triangle) while the sixth group (G5) had difficulty to decide between several alternative classifications for right-angled triangles focusing on their distinctive attributes (the right-angled isosceles triangle, which was correlated with pyramid "like a pyramid", was considered either as a unique representative of a possible subcategory or as the basis for the common classification of all right-angled triangles –the right-angled scalene triangles were also suggested as a separate category; see indicatively excerpt G5A/118). However, during task B, group 5 also suggested their common classification with the rest of the triangles on the basis of the number of sides.

(G5A/118)

X: *Madam, what shall we do with these?* [2 right-angled triangles left apart] *If we make it like this, it looks like..* (Touches the tip corner of a right-angled scalene triangle)

Tch: *Like what?*

X: *Like a triangle I think, but (…) like a fin.*

Kind: *Are we going to leave them separately, is it another group?*

E: *Let's make it to be two, a bit separate* (she removes the two triangles from the main triangle group) *but be alike, not be with another group, so they are alike.*

Rhombus

Despite the suggestion of some children that rhombi could be included in the triangle category, finally a separate category is formed where a rhombus is described:

1) As “two triangles stuck together -G1A/13 (and vice versa, “if we cut the rhombus in half, we get two *pointy* triangles”).

(G1A/13)

F: *Separate, separate, I say. Rhombi don't go with triangles.*

K: *They do, they do, because we have... it picks here, it picks here too... and here it's the same* (he points at the vertexes and at the acute angles in triangles and rhombuses).

F: *Yes, but you can’t* (you shouldn’t), *yes, but they are not the same…normally* (...)

F: *These here, these are two triangles stuck together, not stuck together… it is a rhombus...This is a triangle, right?* (she raises a rhombus and shows the 2 tangent sides to the other children; then she turns it upside down and shows its other 2 sides)*….and another triangle under it... and so it becomes a rhombus. This is only one triangle* (she shows a triangle from K’s groups).

(G3B/76)

M: *when we cut this here* (shows with a motion a diagonal cut on a rhombus on the page), *it will become a pointy triangle.*

2) By enunciation of its typical characteristics: number of angles and sides.

(G2A/163)

M: *I say it's not for a group* (meaning triangles and rhombuses do not belong in the same group), (…) *this one has 3 angles...this one has one, two, three, four...*

The “theorems in action” (Vergnaud, 1996) the children apply in creating the rhombuses' categories are:

1) The rhombus is a «crooked/stretched square».

(G1A/194)

T: *If we turn it like that...*(turns a rhombus sideways so that the two parallel sides be horizontal.

F: *Well, it's a crooked square.*

2) The rhombus is an «inclined» square.

(G2B/47)

M: *The square... it has 4 angles and if we turn it, it becomes another shape.*

Tch: *Which one does it become then?*

M: *It becomes the rhombus. Now, that is easy for me.*

Squares-rectangles

Five groups classify squares and rectangles in separate groups named correspondingly “Square/-es” and “Rectangle /-s” or “Parallelogram” (2 groups of children), while one group (G5) decides to unify rhombi and squares in a common category named “Rhombus” due to similarity with rhombi in attempts of square rotation (while handling shapes they used both terms “square” and “rhombus”). Regarding squares and rectangles, shape categories often correlated focusing on their contrasting characteristics, the base of children’s classification seems to be the square, as in 3 groups of children the most ambiguous rectangle is added to the group of squares (possibly as a marginal representative at the edges of the category). We should also mention that children use the words ‘parallelogram’ and ‘rectangle’ as identical. In Clements et al.’s (1999) study children also tended to accept “long” parallelograms as rectangles.

The inequality of the rectangle's sides is recognised as its basic distinctive characteristic implied or explicitly notified as in excerpt G1B/172.

(G1B/172)

Tch: *Yes. How do we distinguish these by those?* (i.e. squares and rectangles)

F: *I know, because this is a bit longer and the two lines are bigger and the other two smaller* (points at a narrow parallelogram on the page consecutively the horizontal and vertical sides) *and because this one* (square) *has the lines that are equal.*

DISCUSSION - CONCLUSIONS

Though the findings of this research are suggestive and could not be generalized, a general remark is that the children were able reliably to identify shapes. All children used formal names for shape categories (circles, triangles, rhombuses, squares, rectangles /parallelograms) except for the ellipse (described as a mirror or an egg). Children distinguished shapes into two large categories: those without angles (circles and ellipses) and those with angles (triangles, squares and rectangles). They used different criterion (visual and property) for examples and non-examples of shapes negotiating within their group the meaning of shapes’ category. Their positions do not concern stable, irreversible or of the same kind criteria, but issues introduced for further discussion in correlation with the tasks they had been assigned.

The criterion of *angle* and *line* (side), which was more or less directly employed and mentioned by all groups of children in the previous tasks, prevails at the game performance of task D, where shape should be described with no reference to its name. Adjusting to the demands of the task (more typical, school-type descriptions /competitive game), children chose more typical-decontextualized terms like “angles” and “lines”, which they considered more appropriate in the certain context. On the contrary, plenty of “(looks) like...” type metaphors were used in shape descriptions of the more “open” texts of task A and B (e.g “circle looks like o (letter)”, ellipsis is “like an egg”, “like zero”, “like a mirror” etc). This kind of metaphors, which are also found in task A, but prevail at task B, allow for the essential qualities of the shapes, as well as the general communicational-educational goal of the project. We could say that such metaphors constitute an alternative strategy for shape identification, a kind of descriptive definition related to definitions through examples.

The procedure of shape classification children were involved, highlighted two main elements about their knowledge on geometric shapes:

1. Despite the existence of "prototypes" (in the case of triangles), their prototypical visual image was challenged by the presence of non- prototypical triangles and the communicational context. This means that if the context of the activity affects to such an extent children's way of thought, more emphasis must be placed on the kind of activities suggested at this level of education.

2. There is evidence from our study that most children were able to formulate correlations between circle-ellipse, rhombus-square and square-rectangles, a fact, which allows us to advance the hypothesis about the existence of a transitional stage between descriptive and relational level of thinking. If confirmed with further research, this finding could have significant impact for the instruction of geometry in kindergarten.

Apart from confirming the existence of the syncretic level, our study offers plenty of indices favouring our fundamental hypothesis that when preschool children collaborate in the context of a meaningful activity (van Oers, 1998) they can construct meanings that characterize levels of geometric thinking attributed by research to elder children.

NOTES

[1] Giannisi, P. (2010). *Definitions and classifications by kindergarten children in the context of school literacy practices: the case of geometrical shapes.* MA thesis (supervised by Kondyli, M.), University of Patras [in Greek].

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