FINGER COUNTING AND ADDING WITH TOUCH COUNTS

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This paper describes the design of a digital technology focussed on early number sense (especially counting and adding). This "Touch Counts" application (designed for the iPad) takes advantage of the direct mediation through fingers and gesture of the touch screen interface. Through an a priori *analysis, we show how the perceptual and motor aspects of the application can support the development of cardinality. Using a theoretical perspective on the role of technology that is informed by an embodied approach to mathematics thinking and learning, we provide a case study analysis of how a 5 year old child (in kindergarten) gains emergent expertise in producing and transforming numbers.*

Key words: Counting, Kindergarten, touchscreen technology, addition

INTRODUCTION

Current mathematics education software has been developed for the desktop/laptop paradigm of technology use where the mouse and keyboard are essential interfaces. Even software for interactive whiteboards (IWBs) does not take full advantage of touch-screen capacities because the mouse/keyboard interface is the default interaction mode. In addition, IWBs, while providing a social space for interaction, do not allow individual students, or small groups of students, to each interact directly with the software. In contrast, iPad devices permit both whole-class and individual interactions. Also, depending on the application's design, iPads enable collaborative interaction between two or three students on a single device (as it recognizes multiple inputs from different individuals simultaneously), something that computers, with a single mouse, have not been able to offer. As well, their small size (book-sized at ~24 x 19 x 1 cm) overcomes obstacles faced by teachers using school computer labs (e.g., awkwardness of obtrusive monitors in isolating rows of desks).

The touch-screen iPads also enable direct mediation, allowing children to produce and transform objects with fingers and gestures, instead of through a keyboard or a mouse. Recent neuroscience research has shown that there is a neurofunctional link between fingers and number processing, and that finger-based counting may facilitate the establishment of number practices (Andres, Seron, and Olivier 2007; Kaufmann et al. 2008; Sato et al. 2007; Thompson et al. 2004). Research has already shown that consistent use of fingers positively affects the formation of number sense and thus also the development of calculation skills (Gracia-Baffaluy and Noel 2008). This suggests that using the fingers to create numbers in a correctly ordered way, with both visual and auditory feedback can support the development of number sense and provide the foundation for successful arithmetic achievement.

While many number-related applications can be found for the iPad, the large majority of them are designed for game-like interactions in which learners practice arithmetic operations. Although they must touch the screen in order to play these games, they are not using their fingers to create or act on the numbers. Our interest was in developing an expressive technology that supports the development of meanings related to numbers and operations. A similar project has been undertaken by Ladel & Kortenkamp (2011), who have developed a multi-touch-table environment in which children can place a certain number of virtual tokens on the table using their fingers.

DESIGN OF THE *TOUCH COUNTS* INSTRUMENT

We've taken a modular approach to the development of the application, which means that the main app will contain several different sub-applications that are meant to offer an evolving sense of number for the learner. Currently, there are two sub-applications, one for Counting (1, 2, 3, …) and the other for Adding (1+2+3+…). On this opening page, the user can choose the language option (French, English, Italian) as well as options for the functioning of the Counting world.

Counting World (1, 2, 3, …).

In this world, learners tap their fingers on the screen to create small numbered circles that are also represented through both symbol (written numeral) and sound (spoken word) as fingers are placed onscreen. In the default mode, gravity makes these circles fall off the screen, unless they are placed on the horizontal bar (see Figure 1). Adding more fingers continues the counting. Fingers can be placed onscreen all at once to create a group of numbers. So, for example, placing five fingers on the screen creates five numbered circles but produces only the word 'five' orally. If the user repeatedly touches the screen with two fingers, she will see pairs of numbered circles appear but will only hear the even numbers. Every finger touch produces a number; this means that it is not possible to move existing objects on the screen.

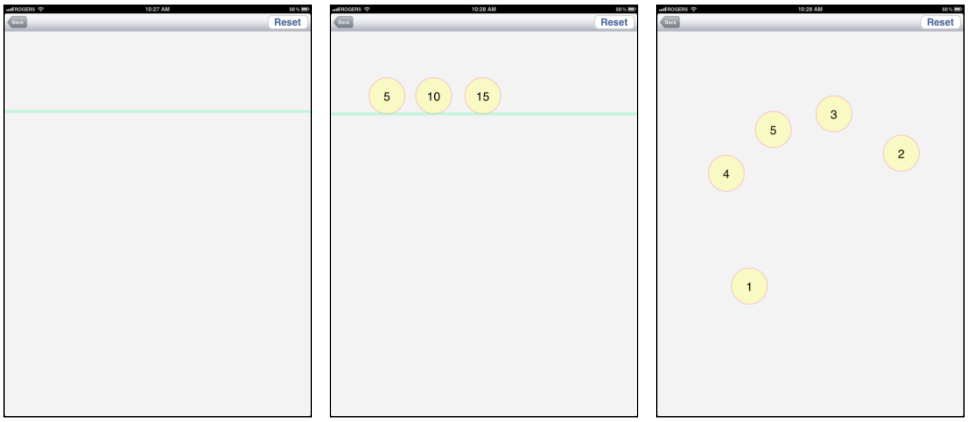

Figure 1: (a) Default Counting world; (b) Default Counting world with numbers on the horizontal bar; (c) No gravity Counting world

The goal of this simple application is to assist young children (grades preK-1) in developing an understanding of the one-to-one relationship between their fingers and numbers. Children at this age, when asked to count, do not necessarily associate the words for each number with the objects being pointed to (this is often called "rote counting"). They tend to recite the numbers as if it were a song and point at the same time, but not always coordinating the two actions (Fuson, 1988). The Counting world should directly supports two of the five aspects of counting identified by Gelman & Meck (1983): (1) when counting, every object gets counted once and only once (one-to-one correspondence principle); (2) the number words should be provides in a constant order when counting. Further, when the gravity option is turned off, it becomes evident that the last number that is counted is the number of items on the screen, which is a third principle of counting. (The other two principles are: that it doesn't matter in which order objects are counted and that it doesn't matter whether the items in the set are identical.)

The horizontal bar was added as an option after observing one 6 year old placed certain numbers (multiples of 10) in one area of the screen, separate from others. Being able to select specific numbers to "pull out" is evidence of having objectified number, that is, of being able to think about a particular number as being more than just an element in the process of counting. More specifically, in order to place a given number on the bar, one must know what the previous number will be. This objectification of number enables the move from ordinality to cardinality, the latter being necessary for answering the "how many?" question.

Nunes and Bryant (2010) have argued that children need to make three types of connections between number words and quantities: "they need to understand cardinality; they need to understand ordinal numbers, and they need to understand the relation between cardinality and addition and subtraction." While the Counting world focuses on ordinality and the objectification of number, the Adding world specifically targets the idea of cardinality.

Adding World (1+2+3, …).

While tapping on the screen in the Counting world creates numbered objects, tapping in the Adding world creates a group of discs labelled by the cardinality of the group (see Figure 2a). Placing five fingers on the screen will create a group of five discs arranged around the circumferences of a circle as well as the numeral 5 in the middle of the circle. This focusses attention on the cardinality of five, rather than its ordinality. As Vergnaud (2008) has argued, understanding cardinality involves more than knowing that the last number in the sequence of counting objects in a set is the number of objects in the set. It involves being able to use numbers in operations and, more specifically, being able to count on. In the Adding world, once two or more sets have been created, they can be added by using a pinch gesture (see Figure 2b). Thus cardinal numbers are ones that can be acted on (in this case, added). Pinching two groups together provides the fundamental metaphor for addition, which is that of gathering together (see Lakoff & Núñez, 2000). The explicit use of gesture in this application is not only based on the affordances of the iPad device, but also draws on recent research highlighting the important relation between gestures and learning (Goldin-Meadows, 2004), and the recommendation that children be exposed to and encouraged to use more gestures (Cook & Goldin-Meadow, 2006, Singer & Goldin-

Meadow, 2005). When two or more groups are added, the discs in them retain the colour of the original sets so that the sum retains a trace of its construction (see Figure 2b). When two or more sets are added, the value of the sum is given orally.

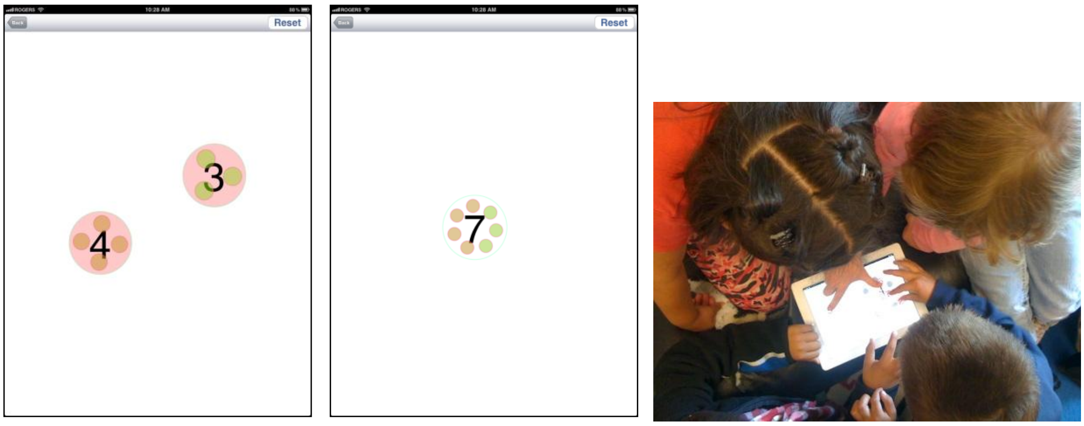

Figure 2: (a) Two groups in the Adding world; (b) resulting sum with colour-based record of the addends; (c) Children using the pinching gesture to add two groups

The Adding world is intended to provide an embodied practice for the addition operation. Children need not know how to add before using this environment. And while a teacher might introduce the word 'adding' to the task, it does not appear on the screen. As such, words such as "making," "putting together," "joining" can all be used to describe the action of pinching sets together. Note that the pinching gesture is symmetric, which means that there is not order implied to the sum of A and B.

For Vergnaud, the student who can answer how many objects are in a set if you add some objects to the set that they have just counted has a sense of cardinality. By having the Adding world groups labelled with their cardinality, the action of counting on is facilitated since the child will focus more on the cardinal number displayed in the group than on the objects in it.

THEORETICAL PERSPECTIVE

Broadly speaking, we situate our work within the area of mathematics education research that examines the role of technology, tools and artefacts in mathematics thinking and learning (see Hoyles & Laborde, 2010). Within this area, given the tangible nature of the interface, we are particularly interest in the embodied practices that allow learners to interact with digital technologies. While other theoretical approaches such as instrumentalism, sociocultural theory and semiotic mediation do not discount the role of the body in mathematical practice, Nermirovsky's (Nemirovsky, Rasmussen, Sweeny, & Wawro, 2012; Nemirovsky, Kelton, Rhodehamel, in press) perceptuomotor integration approach focuses specifically on the way that mathematical expertise develops through a "systematic interpenetration of perceptual and motor aspects of playing *mathematical instruments*" (in press, *emphasis in original*). This approach shares many similarities with the emerging body of work in mathematics education that moves away from a mentalist focus on structures and schemas toward a description of lived experiences in which learners' activities are at once bodily, emotional and interpersonal (Radford, 2009; Roth, 2011).

The perceptuomotor integration approach assumes that mathematical thinking is centrally constituted by bodily activity, which may be more or less overt, and that mathematical learning occurs through a transformation in the lived bodily engagement of a learner in a particular mathematical practice. This approach takes a strong stance toward embodiment, seeing it not just as a precursor or underpinning of mathematical thinking, thereby further promoting a mind/body dualism. Instead, mathematics learning entails transformations in the lived body experience, not just at the primary school age when children interact with physical manipulatives, but for learners of all ages. Thus, taking *Touch Counts* as a mathematical instrument, we will be interested in learners' developing fluency and the concomitant changes in the way they touch, move, talk, gesture, etc.

METHODOLOGY

The interview took place near the end of the kindergarten school year in the resource room of an elementary school located in northern British Columbia. Several children were interviewed, all between the ages of 5 and 6. The interviews protocol was intentionally open-ended since we wanted to see what children would be able to do without specific instruction and what kinds of questions/investigations they would initiate on their own. If the child did not notice particular features/techniques (that the bar keeps numbers from falling down, that the Reset button restarts the counting, that the pinching gesture assembles groups) the interviewer provided an explanation. In addition, the interviewer asked each child, after a period of play, to place a certain number on the bar (usually 5) in the Counting World and to make a group of 7 in the Adding World. Other tasks were given when the child seemed to have exhausted a certain investigation. Our hypothesis in terms of instrumentation was that the children would discover the main features of Touch Counts on their own. In terms of number sense, we hypothesised that the children would like to create big numbers and that both tasks (placing just 5 on the bar and making groups of 7) would be challenging. In this paper, we have chosen to focus on a girl named Katy, who has just recently turned 5. Her teacher described her as one of the weaker students in the class.

RESULTS AND ANALYSIS: ONE-FINGER INSTRUMENT PLAY

We divide the results into two sections, one focussed on the Counting World and the other on the Adding World. For readability, we analyse each section in turn.

Counting World: What kind of number is going to come after?

The session began with the interviewer saying, "Let's start with number." Without any instruction, Katy started by placing her right index finger on the screen and swiping it downward (Figure 3a). She did this slowly, repeating the numbers as she goes (saying some of them out loud, like 2, 3 and mouthing the others). After 9, she put her head down, created a number then repeated 10 out loud (Figure 3b). She lifted her head up at 14 and kept making numbers. At 17, she put several fingers on the screen at once and the iPad said 21. She paused and smiled. She then continued, with her index fingers, to make numbers up to 27, saying the numbers at the same time as the iPad. She looked up, no longer watching the screen and continued swiping and saying numbers (Figure 3c). She had automated her number-making, swiping the screen in a rhythmic way without having to look.

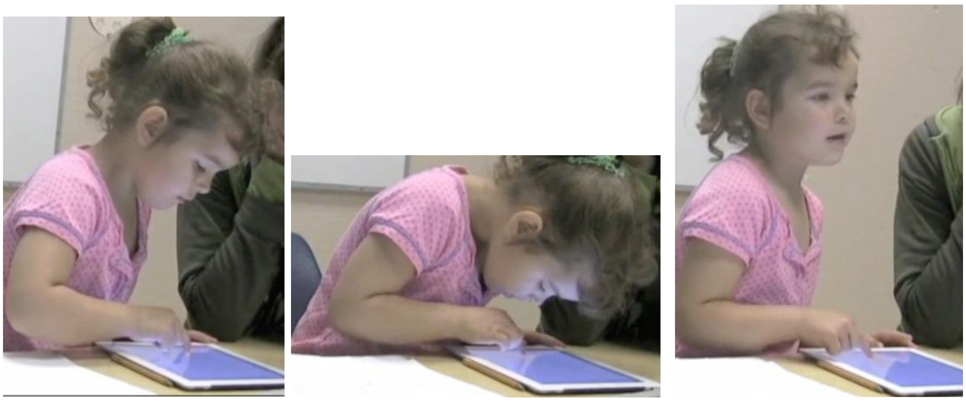

Figure 3: (a) first interaction; (b) bending over around 10; (c) counting beyond 27

After accidentally pressing the Reset button, Katy began counting again. The interviewer then invited her to put numbers above the bar. When she put her next number above the bar, she said, “It stops the number. Why?” She then tapped her index finger more quickly and said, “pop.” She continued putting numbers above the line, saying “pop” each time, and lining them up in a row moving from right to left. She stopped at 47, sat back and smiled. The interviewer asked Katy to press the Reset button. The interviewer then said, “I want to see just five up here.” Katy tapped with her index finger above the line. The interviewer says, “I don’t want to see one.” Katy resets (without being asked). She then put 1 above and 2, 3, and 4 below the bar, and then 5 above. She sat back and smiled. When the interviewer prompts her to put just 5 on the bar, Katy resets, put 1, 2, 3, 4 below the bar, saying four out loud. She paused, then put 5 above the bar, saying five out loud.

During this whole interaction, except for a brief multiple-finger tap (perhaps accidental), Katy used her right index finger. However, the way she touched the screen changed from a slow swipe when she first creating numbers, to a quick tap (“pop”) when she was putting numbers above the line. As she tried to put 5 above the bar, her tapping got slower. When she succeeded in getting only 5 above the line, her tapping became quicker and rhythmic, suggesting that she could anticipate when 5 would come.

The interviewer then asked whether Katy could put 5 and 10 above the line.

40	I:	Imagine five and ten are your best friends and they are the only ones you want to have come over to your house.
41	Katy:	[*Smiles, looks up*]. Okay. [*Sits up in her seat. Taps 1, 2, 3, 4 down and then 5 up. Smiles. Taps 6 down*.] What kind of number is going to come after?

42	I:	After 6? What do you think?
43	Katy:	Don’t know. One, two, three, four, five, six, seven! [*Taps 7 down and then looks up*]. Eight. Does he go there?
44	I:	He’s not your friend, just ten.
45	Katy:	[*Taps 8 down*.] Nine. [*Looks at interviewer*.]
46	I:	Not your friend.
47	Katy:	Is nine going to come after?
48	I:	You just did eight. What do you think?
49	Katy:	One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen. No. [Closes eyes, laughs and tilts her head up. Puts 9 down. Then puts 10 down.]
50	I:	Oh. Ten was our friend.
51	Katy:	[*Resets. Puts 1 up. Resets. Puts 1, 2, 3, down. Pauses. Puts 4 down. Puts 5 down*.]
52	I:	Oops. We lost five.
53	Katy:	[*Resets. Puts 1, 2, 3, 4 down tapping quickly right below the line, up to 5 up. Taps 6, 7, 8 in the same place below the line. Looks up at the interviewer.*] What’s next?
54	I:	After eight?
55	Katy:	Yeah. [*Taps 9 down. Puts 10 up. Smiles and claps*].

Katy tapped the numbers 1 through 4 quickly in a specific area of the screen. In [41], she was pleased to have correctly put five above the bar and then immediately put 6 below. Katy then counted up from 1 to figure out what would follow 6. She used the same strategy after 8, but this time overshooting 9. After mistakenly putting 10 below the bar, Katy reset three times before [53], when she successfully got to 8. She then succeeded in [55] at putting 9 below and 10 above the bar. In [53] her tapping from 1 to 8 had become even quicker, with no pausing before 5.

Hoping to encourage Katy to use more than one finger at a time, the interviewer asked whether she could make many friends at a time. But Katy responded by tapping rhythmically (with one index finger) below then above the bar, up to 98, saying “I’m doing a pattern.” The interviewer repeated the invitation to use more than one finger. Katy put her right hand down (touching the screen also with her palm) and smiled. She created numbers up to 205, then sat up and made more numbers with the left hand, this time with only the fingers (not the whole hand). She switched back to her right hand, pressed Reset with her index finger and said, “I don’t want no friends.”

As Katy engaged with *Touch Counts* through exploration and with the interviewer, she developed several dimensions of tool fluency. Without explicit prompting, she could make numbers on the screen. This was an activity she seemed to enjoy, as she patiently counted higher and higher, repeating the numbers with the iPad. She quickly became adept at pressing the Reset button and placing numbers above the bar. She also became proficient at being able to place a given number n (a best friend) on the bar by tapping below the bar n-1 times and then tapping above the bar. Katy was able to articulate a strategy for deciding where to tap her finger, as evidenced in [41] "What kind of number is come after?" Instead of saying that she has objectified number, we see her actions more in terms of developing local fluency around 5 in that she forges relationship (what's before 5, what's after 5) in the neighbourhood of 5. However, it is evident in her work on trying to get just 5 and 10, that she uses the routine for placing 5 on the bar in order to predict when 10 will come.

The episode shows her ability to pick out a given number developed through perceptuomotor integration. At first, her tapping was slow and irregular, and was often accompanied by her own oral counting, and sometimes by repeating the iPad. But as she made mistakes, hearing and seeing, for example, the number 5 fall off the screen, she would reset on her own and tap anew. The tapping became quicker and more rhythmic until eventually putting 5 on the bar involved tapped her finger four times on a spot below the line and then moving to tap the 5 above the line. We notice too that Katy, despite several prompts from the interviewer, strongly preferred one-finger playing. So, despite the fact that she can "objectify" 5 and 10, we see her as an ordinal Touch Counts player—she creates numbers one at a time.

Adding world: 7 involves making more ones

The interviewer switched to the Adding world. Katy immediately put her left hand on the screen, creating a group of 4, then a group of 5, then 4, then 2. When the interviewer asked her to bring two groups together, Katy uses her right index and middle finger to gather groups of 4 and 2. But when she tried to bring other groups together, Katy inadvertently created new groups. The interviewer showed her how to gather groups with two index fingers. Katy tried this, making a group of 5. She then unintentionally creates a group of 7. The interviewer asked, "Can you make a group of seven for me?" Katy tapped with her index finger several times, then made a group of 4. She then tried gathering the groups together, but ended up creating several more groups of 1 and 2. Seeing that she was having trouble making groups, the interviewer told Katy that she could use two hands. Katy eventually makes a group of 4 (using one hand only). The interviewer asked her again "How could you make seven?"

71	Katy:	Four and one [*pinches groups of 4 and 1 with her right hand thumb and index fingers, making a group of 5*].
72	I:	You have 5 now and how many more do you need to put in there [*pointing to the group of 5*] to get seven?
73	Katy:	One? [*Katy struggles to put to groups together. Gathers the group of 6 and a group of 1*].
74	I:	Six! Oh, you are almost there, you got six.
75	Katy:	[*Makes groups of 1 and a group of 3*.] Not three. No. You go. [*Drags 3 to the corner of the screen.*] I need you one. One? [*Holds 1 with her right hand index finger and gathers to the group of 6 using her thumb*.] Again.

When asked whether she wants to make another group of 7, Katy used her middle and index fingers and she continued to gather more 1s together to make another group of 7. The interviewer asked her to gather a group of 2 and 4 (already on the screen), but she said, “I’m going to make more ones.” She continued to work diligently until the interviewer showed her again how to use her two index fingers to gather groups. She did it herself, made several groups, and then returned to using just one hand.

In moving to the Adding world, Katy experienced difficulty in gathering groups together since her fingers would land on a blank part of the screen instead of on an existing group. At first, her gathering was haphazard, sometimes resulting in a combination of groups but most often in the creation of a new group. She more or less refused to use two hands to make the gathering easier. Nonetheless, she became fluent in gathering two groups, especially when one of the groups was a group of one. Using the strategy of successively adding groups of one, she was able to create several groups of seven. In Vergnaud’s sense, Katy was evincing a sense of cardinality as she was operating with the groups, adding 1 to 1, 2, 3, 4, 5 and 6. However, at no time did Katy intentionally add anything other than a 1 to an existing group in order to produce a group of 7. Indeed, when invited to gather a group of 2 and a group of 4, Katy was insistent on “making more ones.” We hypothesise that adding-on one, in *Touch Counts* at least, offers an intermediary kind of expertise for children like Katy, who are deeply oriented toward ordinality but are using an instrument that expresses cardinality.

CONCLUDING REMARKS

As we saw in the Counting World, there seems to be a close connection between Katy’s preference for one-finger (or one hand) actions and her almost exclusive use of adding-on one. We are interesting in examining the correlation between this kind of one-finger interaction and students’ number sense. In addition, we would like to study whether explicit instrumentation of two or more finger touching (in both the Adding and Counting Worlds) might help develop children’s number sense.

Our goal in this paper has been to describe *Touch Counts* and provide a rationale for its design. In our exploratory study, we have shown that children easily and sometimes spontaneously learn to play this instrument. We have also shown that there may be a strong relationship between the way children actually use their fingers in playing this instrument and the way they think about numbers—a finding that is consonant with the dialogical nature of our theoretical framework. Finally, we have shown that the tasks offered in the two worlds enabled Katy to develop a certain kind of expertise in working with cardinal numbers, which suggests ways of thinking about ordinality and cardinality that are specific to this instrument rather than technology-independent, as suggested in the literature.

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