THE STRUCTURES, GOALS AND PEDAGOGIES OF "VARIATION PROBLEMS" IN THE TOPIC OF ADDITION AND SUBTRACTION OF 0-9 IN CHINESE TEXTBOOKS AND REFERENCE BOOKS

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Different cultures with their own advantages and disadvantages are, rather than oppositional, complementary. In this paper, we attempted to articulate a Chinese "indigenous" pedagogical practice. This "indigenous" practice, "variation problems", in the topic of addition and subtraction of 0-9 in Chinese textbook examples and their goals and pedagogies in their textbook reference books are presented. To grasp its distinctiveness, a comparison between Chinese and American textbooks is carried out. It might enable us to see which parts of the different educational systems can learn from each other.

Keywords: variation, addition and subtraction, Chinese mathematics education, textbook design, textbook reference books

INTRODUCTION

Different curriculum traditions are developed in different cultural communities (for a general discussion, see Xie & Carspecken, 2008). Priority of "contextualization problems" in the interest of facilitating connection situation is generally regarded as the common curricular trend in the West (Clarke, 2006). In contrast with "contextualization problems", variation problems in the interest of facilitating connection concepts and methods play important roles in the eastern curriculum. It is generally perceived as one of the most valuable experiences within Chinese mathematics education community (e.g. Sun, 2011). From outside perspective, Marton (2011) argue that variation practice stems from the Chinese language expression. This study tries to indicate how this practice reflected Chinese own advantages and disadvantages of curriculum. It is interesting to note there is robust literature in textbook comparison in the field of math education. The main direction of textbook comparison focuses on contents and problems. For example, Fuson, Stigler & Bartsch (1988) concentrated on the grade placement of topics, content topic covered and page space of each topic. Li (2000) stressed the "problem" perspective (1) number of steps required; (2) context; (3) response type; and (4) cognitive expectation. However, there are little textbook studies reveal the textbook difference from variation perspective.

Problems with variation

There are two mainstreams of variation practice studies: one developed from the Chinese tradition as local Curriculum design model (e.g. Gu, Huang & Marton, 2004), another evolved from the European tradition, mainly from learning perspective (e.g. Marton and Booth, 1997). Following the European tradition, Watson and Mason

(2005) pointed out two important parameters of mathematics structure: the dimensions of possible variation and the associated ranges of permissible change should be stressed in the use of examples. In this study, we aim to introduce European readers to variation practice from the Chinese tradition. A number of studies (e.g. Gu, Huang, & Marton, 2004; Sun, 2007) consistently claimed that variation practice offers some advantages in Chinese mathematics education. For example, Gu ,Huang, & Marton (2004) argued that, by adopting teaching with variation, even with large classes, students still could actively involve themselves in the process of learning. The "paradox of Chinese learners" might originally be a misperception by Western scholars due to the limitation of their theories. Sun (2007, 2011) presented a Chinese pedagogical phenomenon in organizing a curriculum with an emphasis on discerning relationships through variation approach and argued that there exists an "indigenous" variation practice uniquely rooted in cultural backgrounds. It has been used broadly in example or exercise design to extend the original examples, known widely in a certain way as "one problem multiple solution" (OPMS, 一題多解, varying solutions), "one problem multiple changes" (OPMC, 一題多變, varying conditions and conclusions), and "multiple problems one solution" (MPOS, 多題一解, varying presentations) .This practice is typically regarded as a natural strategy to deepen the understanding in local curriculum as a daily routine, which perhaps makes the "indigenous" practice distinctive. Sun (2011) described these practices and their roles in the topic of fraction division. Reader could further wonder why Chinese textbook authors design them in this way and how to use them. In this study, we explore structures, goals, and pedagogies of problem with variation in the topics of addition and subtraction of 0-9, the most vital and central concept for later mathematic learning.

Addition and subtraction of 0-9

Addition and subtraction of 0-9 is vital and central concepts for later mathematic learning, which would influence numeracy, algorithm understanding of multi-digit addition and subtraction, of multiplication and division, of decimals, of fractions. It is central to developing number sense and is also the basis for the four fundamental operations on numbers and concepts that comprise elementary school mathematics. Not only does it connect to all important concepts, it is also a prerequisite for any real understanding of whole and rational number system". For example, US national performance of subtracting with regrouping of 2-digit is only 28% correct in Grade 2 and their error remains extremely common until high school (Fuson & Li, 1990). Cross-cultural comparison indicated that Chinese teachers have a deeper conceptual understanding of subtraction with regrouping, a solider knowledge of abundant connections and much more flexible way to explain problems than their American colleagues (Ma, 1999). Is it related to their learning resources?

As mentioned before, it seemed that Chinese arithmetic development, textbooks, their textbook reference books, and particular variation practices, might be a good clue for understanding Chinese mathematics education system rarely known outside of Chinese community. To enable us to see which parts of the different educational systems can learn from each other, in this study, we would like to go further to explore structures, goals and pedagogies of "variation problems" in the topic of addition and subtraction of 0-9. For comparison, a USA textbook and their according for teaching guide was chosen as a "mirror", which reflected the curriculum constructed upon different philosophical traditions: Dewey's instrumental pragmatism in the case of USA compared with dialectical materialism in the case of the Chinese mathematics curriculum (Xie & Carspecken , 2008) . The research question of this study is restated as follows: *What are structures, goals, and pedagogies of variation problems in the topics of addition and subtraction of 0-9 in Chinese textbook and reference book?*

A Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005) used for over 30 years by the majority of students composed of diverse backgrounds was chosen, a representation of the Chinese national curriculum by most scholars of textbook comparative study. This textbook with its textbook reference book (Elementary Mathematic Department, 2005) is an authoritative guide for all teachers on what to teach/learn and how to teach/learn as national examination problems are required to be "from textbooks, but above textbooks". An American textbook Teacher edition (similar to Chinese guide book) (Gonsalves, Grace , Altieri, Balka, Day, 2009) identified as a widely used mathematics textbook was chosen to act as a "mirror". Here we translated "indigenous" variation practices into codes by examining "problem set with /without concept connection" or "problem set with /without solution connection" (Sun, 2011). Note that this textbook is not claimed to be representative of all the textbooks used in USA, but we consider that reflect the typical practices in USA.

It is deserved to note textbooks play different roles in their system. Chinese textbooks have been regarded as the most authoritative books in local culture. They have been both driven and governed by government system since Tang dynasty, different from those driven by markets in western culture. Chinese textbooks as textual art of pedagogy are required to rigorously present what teacher should teach and what a student should learn than those in other places. Therefore, they are generally designed by local experts collected in the entire county. They are expected to play multiple functions in Chinese mathematical education system, such as main media for teaching and learning in the classroom, self-learned instrument for out-of-school learners, tools of teachers` professional development by intensively studying textbooks (e.g. Ma, 1999) and its reference series (教學參考書), which is regarded as one of the important professional development notions in China.

THE STRUCTURES, GOALS, AND PEDAGOGIES OF VARIATION PROBLEMS

The Chinese textbook includes 13 examples with problem sets of concept connection (OPMC), in total, accounting for about 87% of all 15 examples. Furthermore, the Chinese textbook includes 2 examples with problem sets of solution connection (OPMS), in total, accounting for about 13.4% of all 15 examples. None of these examples appear in the American textbook. The invariant mathematical meaning of addition and subtraction, that is, part-part-whole concept is the “core” idea highlighted in Chinese textbooks, which are not pointed out in American textbook.

The design of addition and subtraction content of 0-9 are typical, which could reflect the consistent features in other 4 chapters too. In the following, we present the structures, goals, and pedagogies of variation problem in the topics of addition and subtraction of 0-9 in Chinese textbook /reference books and American textbook of student/ teacher edition. We will begin with the structures, goals, and pedagogies of OMPC.

Structures, goals, and pedagogies of OPMC

OPMC in the topics plays two roles: providing foundation and making concept connections. In what follows, these are introduced with illustrative examples of providing foundation. It is interesting to note that addition or subtraction is not introduced directly, but its knowledge foundation as knowing number by OPMC is systematically provided. Fig. 1, 2 shows two examples introducing the quantity concept of 4, 6, 7, called cardinal number, by the problem variations with composition

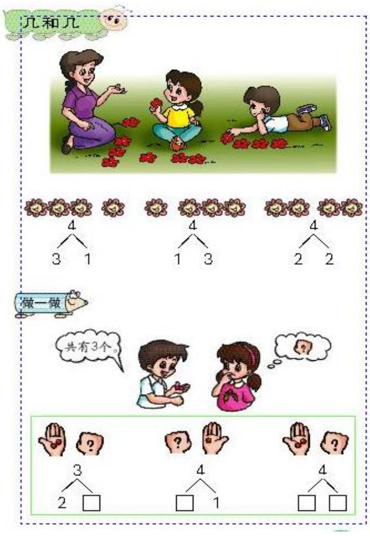

Fig. 1. The example introducing the cardinal number concept of 4 using the problem variation with composition and decomposition concept connection in the Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005, Vol. 1, p.19)

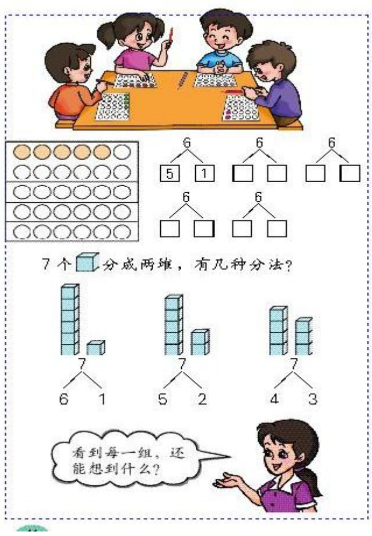

Fig. 2. The example introducing the cardinal number concept of 6,7 using the problem variation with composition and decomposition concept connection in the Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005, Vol. 1, p.44)

and decomposition concept connection in the Chinese textbook.

It is noteworthy that the design is unique that knowing number, concept of addition, and concept of subtraction are united together in all 6 chapters and gradually expand from 0-5, 6-10, 11-20, two-digit, three-digit, above four-digit in the Chinese textbook, which is separated into 20 chapters with tittles of pattern and number sense, that of addition strategy, that of subtraction strategy in the American textbook. Their design goals and pedagogies of figure 1& 2 are explained in the following in its reference book.

> Knowing numbers is the premise of calculation. Conversely, calculation will help to deepen understanding of numbers. For young children, the strategy combining knowing number with basic calculations would be, not only easy for learning number concept, but also conductive to consolidate basic calculations learned inversely. (Elementary Mathematic Department, 2005, P.34)

The goal and pedagogy of figure 2 is explained in the following in Chinese reference book.

> The teaching should follows the following procedure: counting → understanding of the order of number → comparison of two adjacent numbers → writing digit →order of number →composition and decomposition of number. The composition and decomposition of number is the focal point. This arrangement, on one hand reflects the rich meaning of number concept, on the other hand also reflects logical order of knowing number as foundation of basic calculations (Elementary Mathematic Department, 2005, 67).

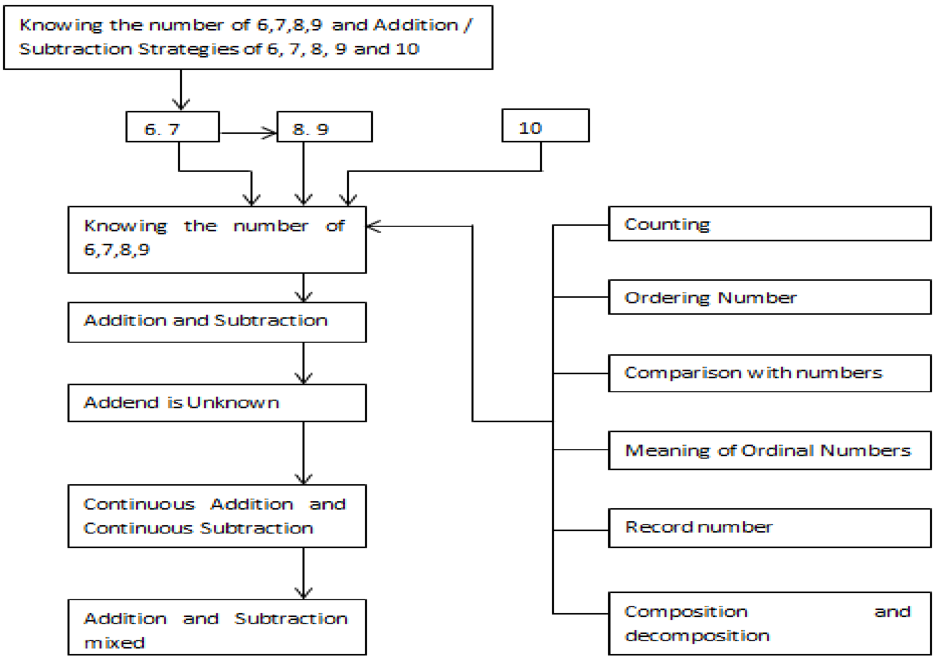

Figure 3.The concept structure of knowing number 6-10 in Chinese teaching reference book (Elementary Mathematic Department, 2005, P.35)

The design above mainly reflect Chinese curriculum tradition with focus on goals – "two bases", namely, the curriculum foundation of addition and subtraction is "part-part-whole" (pre-algebra thinking foundation) relationship. In fact, "two bases" is regarded as the most valuable tradition in the history of Chinese curriculum reform by local experts, different from those in other counties, such as problem solving, communication, reasoning in USA. It is impressive that the concrete foundations, similar to knowledge package (Ma, 1999), in every unit clearly are presented in Chinese teacher guide book. Fig.3 is the concept structure of knowing number 6-10, the concrete curriculum foundation in a unit, in Chinese teaching reference book (Elementary Mathematic Department, 2005, P.35).

It is impressive that two concepts of addition and subtraction are always almost elicited together in term of examples of OPMC in Chinese textbook, rather than separated in the American textbook. In what follows, these are introduced with illustrative OPMC examples of making connections.

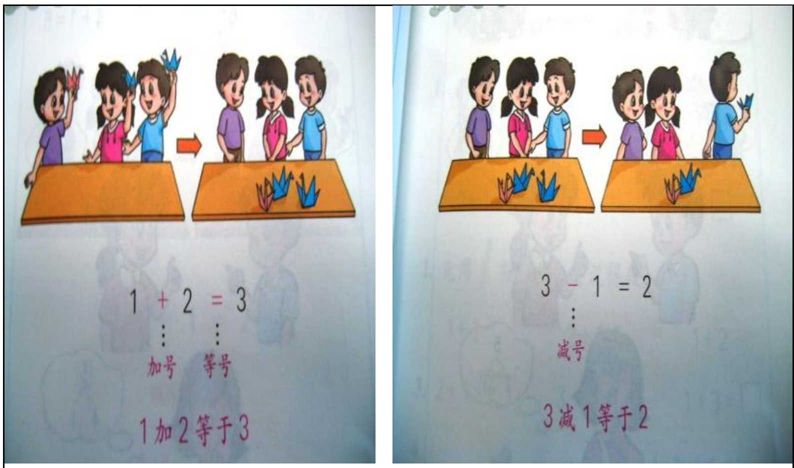

Fig. 4. The example introducing the subtraction concept using the problem variation with concept connection in the Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005, Vol. 1, p.20-21)

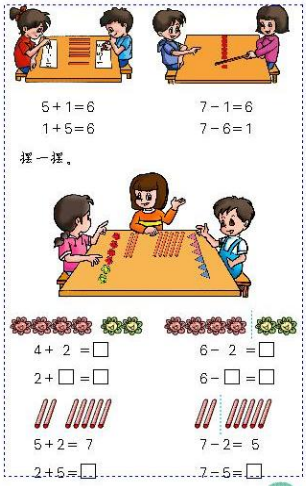

Fig. 5. The example introducing the subtraction concept using the problem variation with concept connection in the Chinese textbook (Mathematics Textbook Developer Group for Elementary School, 2005, Vol. 1, p.45)

Figure 4 shows a paradigmatic example of introducing the subtraction concept by OPMC: 1+2=3, 3-1=2.The problem set intends to help learners to recapitulate the relationship of addition and subtraction, and the meaning of "equal" by three figures, 1, 2, and 3, among the two problems, which may help students to focus on concept variation, rather than digital variation (general feature in the US counterpart). Figure 5 shows a typical example of introducing addition , subtraction , and exchange law of number 6 by three groups of OPMC: 5+1=6, 1+5=6 ; 4+2=6, 2+4=6, 6-2=4,6-4=2 ; 5+2=7, 2+5=7, 7-2=5,7-5=2.The three group of problem set intend to help learners to recapitulate the relationship of addition and subtraction, and the meaning of "equal", which stress the invariant concept of part-part-whole .

The goal and pedagogy of this design is explained below in its reference book.

> The teaching idea of meaning of subtraction is same as that of addition. Textbook use the same situation to elicit subtraction which indicates the relationship that subtraction is the inverse of addition. Therefore, appropriately combining subtraction with addition in teaching will be helpful for students to grasp the relationship and difference of addition and subtraction, which will deepen the understanding of the meaning of addition and subtraction too (Elementary Mathematic Department, 2005, P.39).

Compared with Chinese design, every example in American textbook naturally introduces the concept of addition and subtraction isolated or with limited concept connection as “basic arithmetic facts” such as, “5+7=12” or “12–7=5”) for students simply to memorize (Ma, 1999). In the American textbook, each addition example uses multiple, different, inconsistent concepts, such as “counting”, “combining”, and “adding”. Each subtraction example uses multiple, different, inconsistent concepts, such as “taking away”, “comparing”, “cross out”, and “identifying inverse operation of addition”. The comparisons are weaker than those in Chinese textbook in each circle. The concept of part-part-whole is addressed as modelling subtraction, different from the most central status as the knowledge foundation mentioned above in Chinese textbook.

Structures, goals, and pedagogies of OPMS

It is impressive that multiple-solutions are always almost elicited together in term of examples of OPMS in Chinese textbook, rather than single solution in each example in American textbook. Figure 6 is a typical “prototype” example of OPMS in the Chinese

Fig 6, The example introducing three additions solutions using the problem variation whit solutions connecting (Mathematics Textbook Deveiopers Group for elementary School, 2005, vol.1, p.16)

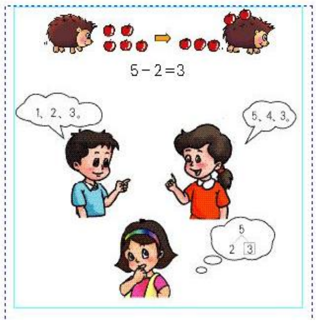

Figure 7, The example introducing three additions solutions using the problem variation whit solutions connecting (Mathematics Textbook Developers Group for elementary School, 2005, vol.1, p.25)

textbook. In the problem variation above, 4+1=5 is designed to naturally introduce a solution system of addition. Within the problem set in the example, there are three solutions given. The first one is that of addition by counting from 1 to 5. The second solution is that of counting from the addend 4 to 5. The third is that of addition by regrouping 5 with 4 and 1. Within the problem set, three addition solutions are presented. Fig. 6 is the problem variation of OPMS above, 5-2=3: The first solution is that of counting what is left from 1 to 3; the second one is counting down 2 from 5(5, 4, 3). The third is that of "separating 5 into 2 and an addend 3 as the unknown. The two group OPMS above intend to help learners to recapitulate the relationship of three solutions of addition / subtraction, and the result of "same", which stress the invariant connection of multiple solutions .

The design goal is explained in the following in its reference book.

> Algorithm diversification is one of the basic philosophies of the "new curriculum standard". It states that: "it is natural students use divertive methods because of different living backgrounds and from different perspectives; teachers should respect their thoughts, to encourage them to think independently, to advocate the diversification. (Elementary Mathematic Department, 2005, P.34)

The design pedagogy is explained as follows in Chinese reference book.

> After students` presentation of multiple solutions, teachers may prompt a discussion on which solution is the simplest one, which help them realize the decomposing-solution is simpler than others. Teacher should guide student from the solution of low level to that of high level. (Elementary Mathematic Department, 2005, P.44)

Compared with Chinese multiple-solution feature, examples in American textbook always elicit the single solution with limited solution connection. It is deserved to note each example in American textbook use many "inconsistent" solutions, such as *"use number line to add"; "doubles"(3 +3 = 6, 5+5=10), "doubles plus 1"(8+9= 8+8+1=17), "compensation*"(*6+8=7+7=14*), and "*reference number*" (*6+7=5+1+5+2=10+3=13, 5 as reference number*) for addition strategy , such as "*counting back"*, " *use of fact families for addition and subtraction*", and *"doubles"*, *"use number line to subtract"* for subtraction strategy emphasizing the importance of applying influenced by Dewey's instrumental pragmatism philosophy (Xie & Carspecken, 2008). Although Chinese textbook authors use multiple solutions, double-number –solution, using-number line-to add, and count-back-solution are not introduced. Only one invariant, "consistent" solution of "decomposing/composing–number-solution" (developed making-a-ten-solution latter) is addressed by OPMS in all the addition /subtraction examples in the chapter, also other chapters with a focus emphasizing the importance of analytical and neopragmatism (Xie & Carspecken, 2008).

DISCUSSION

Many readers may argue that the variation approach may be confusing and that a sequential organization with time gaps ("one-thing-at-the-time") should be preferred. In fact, variation approach might come from different kinds of pedagogical traditions and philosophies developed for centuries. The issue of variations in problem sets directly reflects the old Chinese proverb, "no clarification , no comparison" (沒有比較就沒有鑒別), rather than "to consolidate one topic, or skill, before moving on to another," a notion broadly used in most textbook development (Rowland, 2008) in Europe and throughout the world . It coincidently emanates from the work of Marton, “variation is a necessary condition for effective discernment” as the soul of variation theory. In contrast, this “one-thing-at-the-time” design would clearly provide fewer opportunities for “making connections” compared to those of contemporaneous variation approaches. The “one-thing-at-the-time” design might possibly reflect a hidden conception, making a connection could naturally happen. In this context, the curriculum role of making connections could either be relatively neglected or taken for granted. It is deserved to note, OPMC and OPMS aim to provide opportunities for making connections and further discern, compare the invariant feature of the relationship among concepts and solutions, since comparison is considered the pre-condition to perceive the structures and relationships that may lead to mathematical abstraction. The invariant is repeatedly highlighted by the design as central idea to design Chinese textbook. Conversely, the invariant concept /solution are not stressed after multiple concepts/solutions are presented in USA textbook, which possibly link to fragmentation understanding pointed out by (Ma, 1999).

This study provides the structures, goals, and pedagogies of variation problems, compared with the American system, in the topics of addition and subtraction of 0-9 in Chinese textbook and reference book, which is consistent with the findings on fragmentation in US textbooks and connectedness in Chinese textbooks (Ma,1999). The comparisons above inspire us to develop much more coherence curriculum by addressing knowledge foundation, concept connections, highlight the invariant concepts and solutions in Chinese mathematics education system. Variation approaches may be critical in developing concept-connection curriculum and instruction rarely figured out before. The priority of “contextualization” in the interest of facilitating engagement, motivation, and meaningfulness is regarded as the common curricular trend in the West (Clarke, 2006). In this light, variation problems suggest a way in which way western counterparts could learn from content-orientation curricula in China. Clearly, variation problems are two-edged sword which could lead to more learning challenges compared to “contextualization” problems because they require the use of multiple concepts and solutions targeted. This study inspires us that textbook comparison with their goals (why textbooks are designed in this way) and pedagogies (how textbooks are used) in their textbook reference books would provide a much more *integrated* window for understand curriculum than textbook series alone in the field of textbook comparison (e.g. Fuson & Li, 2009).

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