INITIAL DIAGNOSTIC ASSESSMENT TO SUPPORT LEARNING

Myriam Codes Valcarce, Josefa Perdomo Díaz

Pontifical University of Salamanca, University of La Laguna (Spain)

It is well-known that student learning are influenced by prior conceptions of mathematical elements of a concept. We will focus on the derivative of a function to propose an initial diagnostic assessment to identify them and serving as a basis to introduce the first order ordinary differential equations (ODE) concept. We observed that both the task and the idea-sharing about the students responses, has served them to help rescuing some aspects of the derivative of a function concept that we consider are important to keep in mind to construct the first-order ODE concept.

Keywords: initial diagnostic assessment, prior conceptions, first-order ordinary differential equations, derivative of a function.

INTRODUCTION

From a constructivist point of view the learner uses prior knowledge to build new knowledge; therefore we must pay attention to this key issue (von Glasersful, 1991; Radford, 2008). Hiebert & Carpenter (1992) state that "understanding [a mathematical concept] can be viewed as a process of making connections, or establishing relationships, either between knowledge already internally represented or between existing networks and new information" (p. 80).

Many researchers conclude that prior conceptions of mathematical elements that make up a concept influences the way students understand the concept (Codes, 2010; Sánchez-Matamoros, 2004; Pirie & Kieren, 1992). Therefore, teachers must be aware of "the relevant prior knowledge students will have at the beginning of a learning unit" to plan powerful learning environments (Stern, Hardy, Jonen, Möller & Staub, 2003). A written test is usually carried out (Perdomo-Díaz, Camacho-Machín & Santos-Trigo, 2011).

We are involved on a research project to design a teaching proposal to introduce the first-order ODE concept on first course University that supports the students to establish links among mathematical elements to improve meaningful learning. We select the introduction of this concept by two main reasons: (i) ODE are taught in most of the scientific-technological degree courses due to their applications on several contexts and (ii) it is a concept that leads to cognitive difficulties.

These difficulties are related to different facts, for example, the ODE's solutions are functions instead of numbers (Rasmussen, 2001) or that specific methods for ODE solving hide the relationship between ODE and the concept of derivative of a function (Camacho-Machín, Perdomo-Díaz & Santos-Trigo, 2012a). Other difficulties are related to the use of graphic representations to explore meanings and mathematical relations (Camacho-Machín, Perdomo-Díaz & Santos-Trigo, 2012b; Habre, 2000). Most of the difficulties seem to be related to the concept understanding of function and derivative and how they are related to ODE. So it is essential to know the nature of the students conception of these mathematical elements that underlie the construction of ODE to plan a good teaching strategy (Stern et al., 2003). We advance that first-order ODE concept is built from three main mathematical concepts: equation, function and derivative of a function.

From the literature review made in Perdomo (2010), we conclude that there is a lack of works emphasizing the relevance of prior conceptions of the students about the concepts of equation, function and derivative of a function, when acquiring the first-order ODE concept. As a result, we present in this paper original work focused on the student's prior conceptions about the derivative of a function. We have prepared a questionnaire to introduce the first-order ODE topic as the initial diagnostic assessment. The class discussion will bring on some derivative relevant aspects.

This proposal is useful both to the teacher and the students. Knowing the prior students' conceptions makes teacher's work easier because provides information to raise a teaching strategy based on the knowledge that is available to the students, and can correct misconceptions. For the students, their prior conceptions make them easier to be awareness of what they know and what they does not, and what their mistakes are. This is the first step to build new knowledge.

THE INITIAL DIAGNOSTIC ASSESSMENT

One of the teacher's tasks is aiding the students to put in the foreground of his mind some cognitive structures related to the new knowledge. One way of achieving this objective is to plan an assessment to give the teacher the prior conceptions of their students. Casanova (1995) defines the *initial assessment* as a task carried out at the beginning of an evaluation process to detect the situation of departure of the students (pp. 75-76). Socas (1997) defines the *diagnostic assessment* as a set of learning situations designed to identify specific learning difficulties; it can be used to determine nature of these learning difficulties. Although some authors equate the diagnostic assessment with the initial (Rodríguez et al., 2000) in short, the assessment's final phase requires the teacher to make a decision, either the promotion of a student, the teaching unit, or the adaptation of teaching strategies and contents to overcome learning difficulties. The assessment process has as objective to take measures to improve student learning (Casanova, 1995).

We propose an assessment that is both initial and diagnostic, since it is carried out before introducing the new mathematical concept (first-order ODE) and its aim is to detect possible problems with the concepts of equation, function and derivative that can become into learning errors. We also add another goal: help students to put in the foreground of his mind some mathematical elements that make up the first-order ODE concept.

Before designing the activities that would be used to introduce the concept of ordinary differential equation, we constructed a conceptual map with the relationship among first-order ODE and three main mathematical concepts: equation, function and derivative. Essentially, a first-order ODE is an equation where the variable is a function of a real variable; this equation set up a relationship between a function that model a process and its derivative that models the process variation.

Activities were designed according to the connections established in the conceptual map. We prepared three main groups of activities that we term as *Questionnaire of prior knowledge*, *Newton law of cooling* and *Mathematical context*. In this paper we discuss the part of *Questionnaire of prior knowledge* activities regarding the differential function student conceptions.

FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS

Different constructivist perspectives agree on the need for linkages assembling previous mathematical elements or constructs to generate new knowledge (Dubinsky & McDonald, 2001; Ron, Dreyfus & Hershkowitz, 2010). We understand that a student learns when he establishes links among different parts of a concept and among different modes of representation (Duval, 1993). The quantity and quality of these links are directly related to the student understanding level (Codes, 2010; Hiebert & Carpenter, 1992; Sánchez-Matamoros, 2004). Taking into account that the foundations of the new knowledge are formed by relatively more basic concepts, we believe it is necessary to pay our attention on them. In Perdomo-Díaz, Camacho-Machín & Santos-Trigo (2011) most students connect the derivative with only two or three different meanings, mainly slope, rate and monotonicity.

The design of our teaching experiment begins with a reflection on what students know about the mathematics elements that are linked to first-order ODE. Furthermore, we create a scenario of reflection, inquiry, argumentation and problem solving using the interaction among students, as an element that promotes them and increase their cognitive potential (Cobo & Fortuny, 2000).

We propose that learning first-order ODE needs the topics equation, function, and derivative to be connecting. Related to the last element, it is needed to consider it as a function resulting from the transforming of another function. Moreover, it must be linked with the slope of the tangent to a function at a point, and with variation phenomena (Perdomo, 2010).

We agree with Socas (1997) on the need to review the source of errors from the point view of both the difficulties inherent in the mathematics and those arising from the teaching and learning process. Socas advocates a training sequence in two phases: a brief assessment to identify the student errors and difficulties, and later phase to introduce the new concept. After the assessment phase, the teacher can design a prevention plan to help the student to overcome the difficulties. We also support the use of the manifestation of errors as a motivating element in the processes of teaching and learning.

COLLECTED DATA

We experienced with the entire group of fifteen first year computer science students that had never listened about ordinary differential equations. Students worked on these activities during a class session of fifty minutes. First of all, we gave them the statement of the activities and led them time to work individually. Finally, there was a whole group discussion about the responses they gave.

The whole group discussion was video-taped, and the student written materials were collected.

The students must answer the following questions:

3.1 What do you know about the derivative of a function? Show an example.

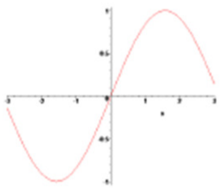
3.2 Use mathematics to express the next situations, and point out if they are regarding the derivative of a function or not.

a) The number of individuals in a population that grows exponentially.

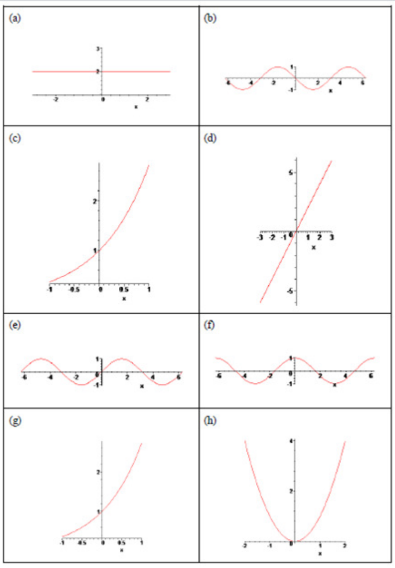
b) The amount of fish in a lake that grow at constant speed.

c) The slope of the tangent to a function at each point in the domain of the function.

3.3 Given the function whose graph is shown below, what can you tell about its derivative?



3.4 Given the following graphs, set couples $\left(f(x), f'(x)\right)$.



With these questions we can achieve two aims: show the teacher which meanings, of the derivative of a function, students have at the beginning of the lesson, and help students to bring them back. We expect that the meaning of slope, variation and function transform come to light.

RESULTS

Question 3.1: the derivative function in the prior questionnaire

To response the first question, the teacher insisted that they must write what came into their head when they heard "the derivative of a function".

Most of the students expressed a conception of the derivative of a function as a procedure and in some cases students related it to the inverse of the integration process.

Student: I can remember that it was the inverse of the integration.

Some of them only wrote an example with a polynomial function:

(Figure 1translation: “Given a function and applying some rules, we obtain another different function”)

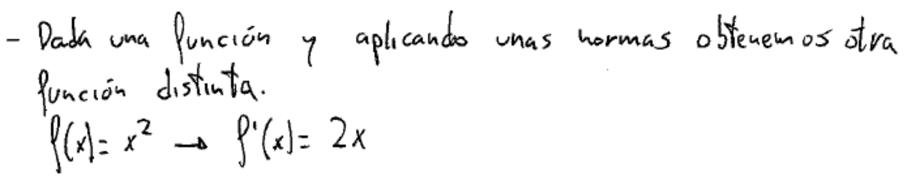


Fig. 1: The derivative function conception as a process.

Two students related the concept of derivative with the slope of the tangent to a function at a point. One of them referred to the derivative of a function at a point, and the other referred it as a function:

(Figure 2 translation: “The derivative at a point represents the slope of the tangent to the function at this point”)

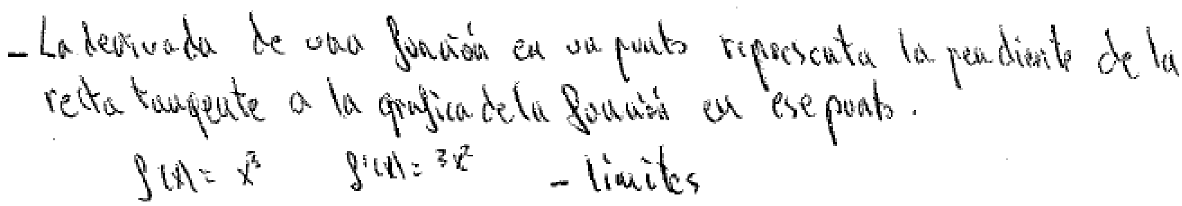


Fig. 2: The differential function conception as the slope of the tangent to a function at a point.

(Figure 3 translation: “It’s a function that denotes the slope of the tangent to the original function. It is the inverse integral operation”)

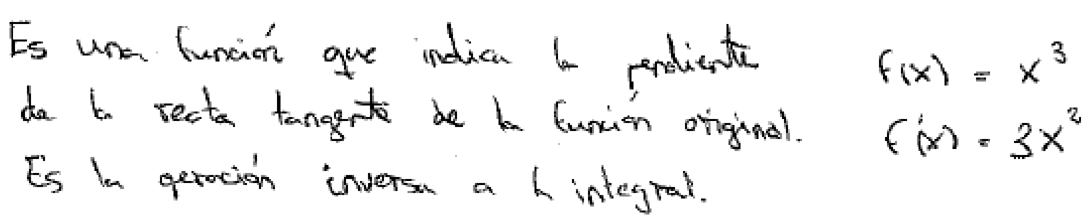


Fig. 3: The differential function conception as the slope of the tangent to a function.

Only one student made reference to the derivative of a function as a process of change.

(Figure 4 translation: "The derivative of a function represents the change and variation respecting to the value of the variable")

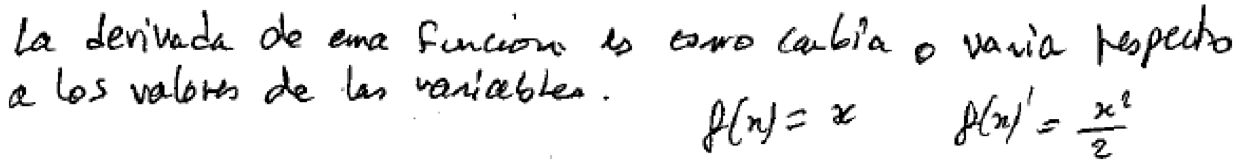


Fig. 4: The differential's conceptions function as a change process.

Question 3.2: the derivative of a function and models

Although all the situations raised in this activity are in some way related to the derivative of a function, few students point out if they were regarding (or not) with a differential function. Most of them only responded with the function that models the situation:

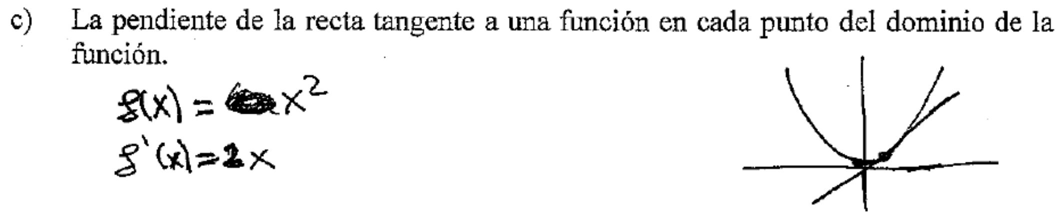


Fig. 5: Some student responses in 3.2 c

In 3.2 a, the student who related the derivative as a change process in the previous question, said that this situation was related to the derivative of a function because a process of change of a variable respect to another one takes place:

Student: There is a change, there is a variable. (…) There is a factor there that varies, good according time no, but according other parameters. (...) For that reason it is, it is a derivative.

In 3.2 b, no student was able to link the linear function $f(x) = a \cdot x + c$ with the statement "grows at constant speed", even visualizing the line $a \cdot x + c$ with $a > 0$ in Cartesian plane.

In 3.2 c only three students draw a curve and its tangent:

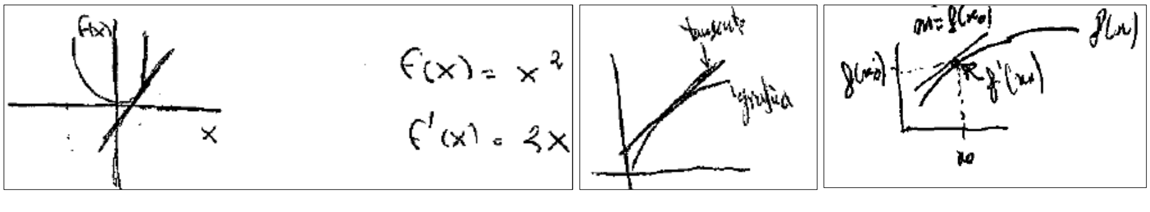


Fig. 6: The parabola $y = x^2$ and two curves with its tangent at a point

One of them said “the slope of the tangent at that point is the derivative of the function at the point”. None of them was able to verbalize that the mathematical representation of the situation was the derivative of a function.

Question 3.3: some derivative properties

In 3.3 most students don’t response the question. During the whole class discussion, several students said that it was the sine function and mentioned the function increasing/decreasing intervals, but no student was able to connect it with the sign of the derivative

Teacher: (…) the function increases. And what can we say about the derivative there?

(Nobody response)

Finally the teacher gave the answer.

Question 3.4: the derivative of a function versus the primitive function

Some students gave right pairs and sometimes one of them omitted a couple, or swapped the couple order $\left(f(x), f'(x)\right)$. Some of these errors are probably due to a distraction.

DISCUSSION

As we mentioned above, it’s important to know the prior student conception about the mathematics elements that are linked to the ordinary differential equation concept. Students should construct a derivative of a function schema with almost three links, even if it’s always available: a function resulting from another function transformation, a model of a process of change and the slope of the tangent to a function at a point (Perdomo, 2010).

Most of the students expressed a derivative of a function idea as a process, in some cases linked to the inverse of the integral. This fact shouldn’t have a negative connotation. The problem arises when students are not able to connect the derivative concept to any other aspect, because the strong algorithmic nature of the mathematical analysis (Artigue, 1991) could manifest as a didactic obstacle (Socas, 1997) for the correct acquisition of ODE concept. In question 3.2 (a) and (b), the students manifested this obstacle when they were not able to explain the relationship between the real situations in which there was a process of change and the concept of the derivative.

The same happen in section 3.2 (c). Moreover, in this section some students expressed the relationship between the derivative and the slope of the tangent only as a local property, nor as a function.

The lack of responses in 3.3 shows a disconnection between the derivative of a function concept and the graphic representation. It’s also a consequence of the mathematical analysis algebraization. However, question 3.4 about the pair

$(f(x), f'(x))$ using graphical representation, got good results. These results suggest a basic knowledge of the graphic representation of some of the main analytical and transcendental functions, and its derivatives.

Justification for the contrast between success in responses to sections 3.3 and 3.4 can be found in the meaning of the derivative that come into play in each of them. To answer 3.3, it is necessary to reflect the relationship between the monotony of a function and the sign of its derivative; in this section, the conception of function as a model for a process of change plays an important role. On the other hand, to answer 3.4 it's enough to bear the graphic representation of some elementary functions in mind and memorize the derivatives rules.

CONCLUSIONS

The activities showed in this document were used to introduce the concept of first-order ordinary differential equation in a first course degree on computer science. Information obtained from the questionnaire helps the teacher to know students weakness about derivate.

On the other hand, although the majority of our students have not built a powerful derivative schema in previous courses to support the construction of the new concept, the idea-sharing about the students responses to initial diagnostic evaluation has served to help them to rescue some aspects of the derivative of a function concept that are important to keep in mind to construct the first-order ODE concept. The activities make the students to reflect on those aspects of the derivative schema that Perdomo (2010) proposed. Sometimes the student simply should remember, while in other cases should re-discover some ideas and add them to the existing. We must consider that this does not ensure the success all the cases, but some students take advantage of these opportunities.

The analysis of the responses to ODE activities will allow us to determine the full scope of initial diagnostic assessment on the students understanding of ODE concept.

NOTE

This work was partially supported by Grant no. EDU 2008-05254 of the National Research Plan I+D+I of the Spanish Ministry of Science and Innovation.

REFERENCES

Artigue, M. (1991). Analysis. In D. Tall (Ed.) *Advanced Mathematical Thinking* (pp. 167-198). Dordrecht: Kluwer Academic Publishers.

Camacho-Machín, M., Perdomo-Díaz, J., & Santos-Trigo, M. (2012a). An Exploration of Students´ Conceptual Knowledge Built in a First Ordinary Differential Equation Course (Part I). *The Teaching of Mathematics, 15*(1), 1-20.

Camacho-Machín, M., Perdomo-Díaz, J., & Santos-Trigo, M. (2012b). An Exploration of Students´ Conceptual Knowledge Built in a First Ordinary Differential Equation Course (Part II). *The Teaching of Mathematics*, *15*(2).

Casanova, A. (1995). *Manual de evaluación educativa*. Madrid: La Muralla.

Codes, M. (2010). *Análisis de la comprensión de los conceptos de serie numérica y su convergencia en estudiantes de primer curso de universidad utilizando un entorno computacional*. (Doctoral dissertation, Universidad de Salamanca, Spain). Retrieved from http://hdl.handle.net/10366/76452

Cobo, P., & Fortuny, J. (2000). Social Interactions and Cognitive Effects in Contexts of Area-Comparison Problem Solving. *Educational Studies in Mathematics, 42*, 115-140.

Dubinsky, E., & McDonald, M. A. (2001). APOS: a constructivist theory of learning in undergraduate mathematics education research. En D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, A. Schoenfeld (Eds.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 275-282). Dordrecht: Kluwer Academic Publishers.

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la penseé. *Annales de Didactique et de Sciences Cognitives, 5*, IREM de Strasbourg 37-65.

von Glasersful, E. (1991). A Constructivist's View of Learning and Teaching. In: R. Duit, F. Goldberg, H. Niedderer (Eds.) *Research in physics learning: Theoretical issues and empirical studies. Proceedings of an international workshop* (pp. 29–39). Kiel, Germany: IPN.

Habre, S. (2000). Exploring Students' Strategies to Solve Ordinary Differential Equations in a Reformed Setting. *Journal of Mathematical Behavior, 18*(4), 455-472.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning. A project of the NCTM* (pp. 65-97).

Perdomo-Díaz, J. (2010). *Construcción del concepto de Ecuación Diferencial Ordinaria en escenarios de Resolución de Problemas*. (Doctoral dissertation). Universidad de La Laguna, Spain.

Perdomo-Díaz, J., Camacho-Machín, M., & Santos-Trigo, M. (2011, March). Some meanings of the derivative of a function. *Poster session presented at the annual meeting of the Seventh Congress of the European Society for Research in Mathematics Education (CERME)*. Poland.

Pirie, S., & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics, 23*, 505-528.

Radford, L. (2008). Theories in Mathematics Education: A Brief Inquiry into their Conceptual Differences. *Working Paper. Prepared for the ICMI Survey Team 7. The notion and role of theory in mathematics education research.*

Rasmussen, C. (2001). New directions in differential equations. A framework for interpreting students' understandings and difficulties. *Journal of Mathematical Behavior, 20*, 55-87.

Rodríguez, T., Álvarez, L., Cadrecha, M. A., Hernández, J., Luengo, M. A., Muñiz, J.,… Soler, E. (2000). *La evaluación en el aula*. Spain: Nobel.

Ron, G., Dreyfus, T., & Hershkowitz, R. (2010). Partially correct constructs illuminate students' inconsistent answers. *Educational Studies in Mathematics, 75*, 65-87.

Sánchez-Matamoros, G. (2004). *Análisis de la comprensión en los alumnos de Bachillerato y primer curso de Universidad sobre la noción de derivada. (Desarrollo del concepto).* (Doctoral dissertation). Universidad de Sevilla, Spain.

Socas, M. M. (1997). Dificultades, obstáculos y errores en el aprendizaje de las matemáticas en la educación secundaria. In L. Rico, E. Castro, M. Coriat, L. Puig, M. Sierra, & M. Socas (Eds.), *La educación matemática en la enseñanza secundaria* (pp.125-154). Barcelona: ICE Universitat de Barcelona- Horsori.

Stern, E., Hardy, I., Jonen, A., Möller, K., & Staub, F. (March 2003). *How to acknowledge cognitive constructivist principles in designing powerful learning environments*. Panel given at the DFG-NSF Conference. International workshop on Research and development in mathematics and science education. Kiel, Germany.