ON-LINE DISCUSSIONS ABOUT EMERGING MATHEMATICAL IDEAS

Chronis Kynigos (kynigos@ppp.uoa.gr) & Foteini Moustaki (fotmous@ppp.uoa.gr)

Educational Technology Lab, School of Philosophy, Department of Pedagogy

Collaborative knowledge-building within an on-line community of learners has for several years been at the core of several studies both for the CSCL and the technology-enhanced exploratory learning approaches. In the Metafora Project, Exploratory Learning Environments (ELEs) are combined with CSCL tools, all in one web Platform. In this paper, we study how the students' meaning making processes were shaped by their on-line discussions in the Platform as they interact with a half-baked microworld. As the Project moves further from the social aspects of learning and touches socio-metacognitive ones, the focus shifts to how on-line learners learn how to learn with and from each other. Thus, we attempt to also study if and how students' meaning making processes are influenced by learning to learn together aspects that come forward as they share and discuss their ideas.

EXPLORATORY LEARNING AND CSCL ENVIRONMENTS

The idea of discussing and sharing artefacts within a community of users through emails, fora or repositories (and nowadays through Web2.0 tools), has long been an issue for technology-enhanced exploratory learning (Resnick, 1996). One of the explicit attempts to develop specific technologies to support the social aspects of exploratory learning was made some years ago through a Project called "WebLabs" (http://www.lkl.ac.uk/kscope/weblabs/index.htm). At this Project, Noss and Hoyles studied groups of students as they collaborated through a web-based system that allowed them not only to share their experiences in a textual form, but also to co-construct and share working animated models (Mor et al., 2006). Apart from the exploratory activities in which the students engaged in their attempt to make sense of "how does this model work", emphasis was put at this Project also on the social interactions among the members of the community and the ways these interactions shaped mutually constructed artefacts. In this case (as in others before), mathematical meaning making was viewed as a process also taking place when students shared and discussed their ideas, argued about the validity of their models, reflected and redesigned their constructions, while working together in groups.

Collaborative "knowledge-building in communities" (Bereiter, 2002) has also been at the core of studies regarding the use of CSCL environments. Stahl (2009) at the Virtual Math Teams (VMT) Project studied small groups of students meeting in chat rooms to discuss on-line about their ideas as they worked with complex mathematical problems. The design of the system also included a shared whiteboard, a wiki for the common artefacts and a portal for social networking. The Project focused on the students' "knowledge-building" processes as a result of their in-group

interactions (what is termed in the Project as “group cognition”), while working with this system. The social practices emerging as the students worked together were considered to be crucial not only for making sense of what they were jointly doing, but also for making sense of how to work together as a group. Thus, the effective collaboration was defined as the one in which the students not only “produced knowledge artefacts” to give to the broader community, but also the one in which each member makes sure that everyone in the group understands and progresses as they should.

In the Metafora Project, we attempt to study students’ mathematical meaning-making as they work in groups with Exploratory Learning Environments (microworlds) together with CSCL tools, all integrated in an on-line Platform (Mavrikis et al, 2012). For this study, two of the Metafora System’s tools were used: the “Twisted Rectangle” half-baked microworld (Kynigos, 2007) and the LASAD discussion tool.

The “Twisted Rectangle” is a 3d Turtle Geometry microworld that includes a buggy procedure. This procedure causes the rectangle that appears on screen to have one of its segments twisted along a plane vertical to the one it belongs to when it's not twisted. It is a half-baked microworld (Kynigos, 2007) in the sense that it holds an interesting idea, but it is incomplete by design, challenging students to deconstruct it and make sense of the reasons for its buggy behaviour. These microworlds have been perceived as 'boundary objects' (Kynigos, 2007) i.e. questionable and improvable objects engaging members of communities in meaning making emerging from the joint de-bugging effort. Thus, they may operate as a tool, around which, the members of the community structure their activities. In this case, meaning generation processes are considered to emerge and be shaped both by the students’ mathematical activity as they interact with the half-baked microworld and their social activity as they discuss on how to make it work, change and customize it. LASAD, in our study, is the tool that allows on-line communication and collaboration among the members of the community.

The Project, however, brings a new strand to integrating ELEs with CSCL tools as it views computer-supported learning in groups as a complex task that requires from students -as they collaborate- to also become aware of elements considered to be important for successful learning in collectives and to learn how to put those elements in use (Wegerif & Yang, 2011). Thus, the group members need to be able to show distributed leadership, plan and coordinate the tasks to be carried out, motivate one another, ensure everybody engages, reflect on the quality of the work through peer reviewing and reflect on the overall progress of the groupwork (Wegerif et al., 2012). All those elements constitute the key components of the “learning to learn together – L2L2” pedagogical approach adopted by the Project.

In this paper, we put emphasis on how the students’ mathematical activity as they interacted with the half-baked microworld was specifically shaped: a) by their need to explicitly articulate their own ideas so as to share them through a discussion tool

and b) by the ideas brought at the table by the other group members. Moving between on-line group discussions and microworld actions, we seek to identify manifestations of L2L2 skills such as organizing and coordinating the work so as to proceed as a group, discussing and evaluate findings from others, reflecting on own findings.

THE DIGITAL TOOLS

The 3d Math Authoring Tool – The Twisted Rectangle microworld

The "3d Math" Authoring Tool (http://etl.ppp.uoa.gr/malt) is a constructionist environment (Harel & Papert, 1991) that allows the creation, exploration and dynamic manipulation of 3d geometrical objects through the use of Variation Tools (Kynigos & Psycharis, 2003). Building and manipulating geometrical objects in 3d Math is not restricted in solely looking at the 3d world form static 2d views. A Camera Controller gives students the opportunity to navigate around, inside and through their constructions, offering the potential for new ways of visualizing 3d space and conceptualizing mathematical notions, especially ones related to stereometry (Moustaki & Kynigos, 2011). We view 3d Math as an authoring tool for developing half-baked microworlds, such as the "Twisted Rectangle" (Figure 1).

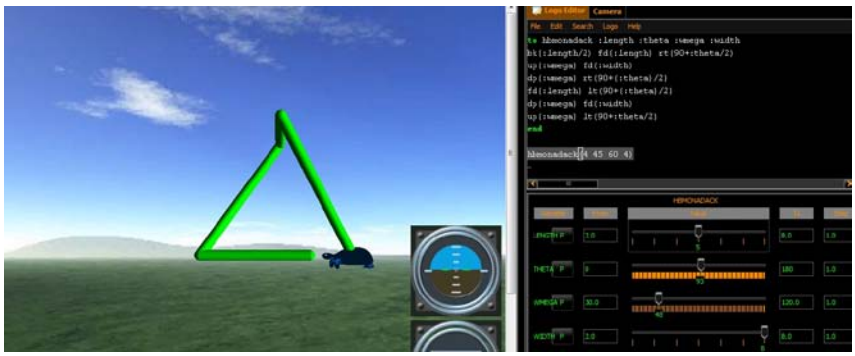


Figure 1: The Twisted Rectangle half-baked microworld in 3d Math

The METAFORA System and LASAD

Being a completely web-based environment, 3d Math is fully embedded in the Metafora System (Dragon et al., submitted), an on-line software platform that offers a set of Exploratory Learning Environments (microworlds) as well as shared workspaces that allow communication among individuals or groups of students. One of those shared workspaces is the LASAD Discussion and Argumentation Tool. The users place in LASAD's UI text boxes with their ideas (we call those "contributions") and link them with existing ones, forming in this way a kind of a structured discussion map (Figure 4). To further tag each contribution with respect to its content (e.g. tag a contribution as "a suggestion" or as "a claim"), a dropdown list is available for each text box. LASAD is designed to function as a tool in which the students may discuss, argue, negotiate their ideas and as a reflection space as it depicts how the Group's discussions evolved.

RESEARCH DESIGN AND METHODOLOGY

Our research approach was based on the idea of studying learning in authentic settings through "design experiments" (Cobb et al. 2003). "Design experiments" aim to contribute to the development of grounded theories on "how learning works" and are conducted with the intention to shed light on the relationships between the material designed for the experiment (usually innovative technological artefacts having added pedagogical value) and the learning processes within a specific context of implementation.

Context and participants

The study described in this paper took place in a Secondary Vocational Education School in a small island near Athens (1st Vocational High School of Salamina) with four 10th grade students (15 years old). The students worked together for 8 hours (2 sessions) in two types of social orchestrations: all four of them as members of just one Group in face to face meetings, and divided in two Subgroups of two members each when working on-line with the microworld and the LASAD Discussion Tool. The researchers adopting a "participant observation" methodology, chose not to intervene in the experimentation to give out specific instructions or to provide the "correct answer" to the students on how to address the challenge and proceed. They preferred to pose meaningful -often intriguing- questions at certain time points, so as to encourage students to continue their explorations, elaborate more on their thoughts, share and discuss their ideas collaborating with the other students. The reseachers in this study had a dual role as they also acted as the class teachers.

Tools and Tasks

Phase 1: Making one side double the other

For this Phase of the Study, we designed in 3d Math the "L" letter microworld (Figure 2a). The microworld's 4-line Logo program includes two variables (one for each of the "L"'s sides). The shape can be dynamically manipulated using the Variation Tool, which allows attributing sequential values to those two variables. The challenge that the students had to address working in two Subgroups (Subgroup A and Subgroup B) was to "make the vertical line be twice as long as the horizontal one", so that the "L" shape changes proportionally as one letter-shape.

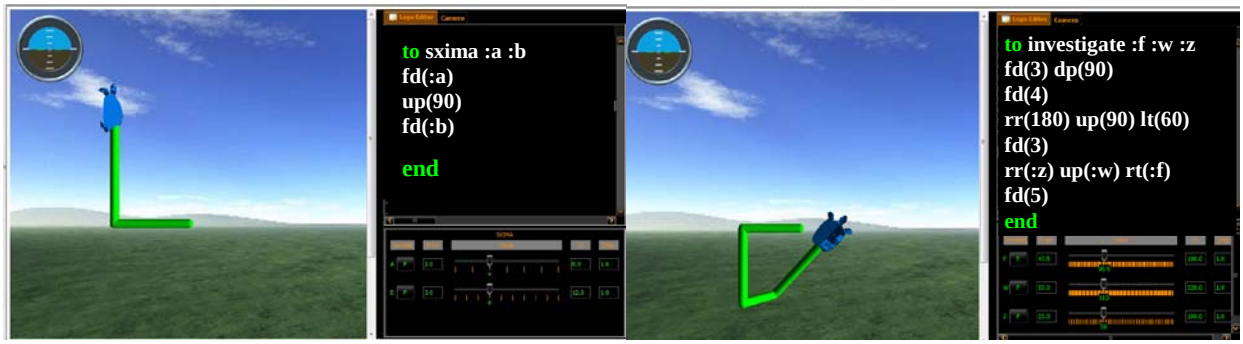


Figure 2a and 2b: The microworlds for Phases 1 and 2

Phase 2: Closing an open shape

For the Main Phase of the Study we designed a half-baked microworld (Kynigos, 2007), called the "Twisted Rectangle". The Twisted Rectangle is a skewed quadrilateral as one of its segments twists along a plane vertical to the one it belongs to when it's not twisted. In this version of the Twisted Rectangle, when running the Logo program, the figure depicted is not a closed shape, but an open one, as the end of one of the rectangle's sides, is not attached to the rest of the shape (Figure 2b). The students working in two Subgroups of two students each (Subgroup A and Subgroup B), were asked to try to "make the shape close". Since, we didn't intend to provide an answer on how to work with variables to do so, but ask them to discuss any ideas within their Subgroup and with the other Subgroup, we had prepared a discussion space in LASAD in which the two Subgroups could meet and share their findings as they explored this issue within the Twisted Rectangle microworld.

DATA COLLECTION-METHOD OF ANALYSIS

A screen-capture software (HyperCam2) was used to record students' interactions the Metafora Tools together with their verbal interactions. Since previous work with 3d Math had shown an extensive use of gestures as means to explain and communicate turtle movements and turns, a Camera was added to record students' hand and body movements. The corpus data is completed by the students' LASAD maps and the Researchers' Fields notes. The video-recorded data from the screen-capture software were verbatim transcribed, while the rest of the data were used for providing additional details. In analysing the data, we searched for verbal exchanges between the students and interactions with 3d Math and through LASAD that indicated that learning to learn together aspects were brought forth as they students attempted to address the challenge when working with the half-baked microworld.

RESULTS

The episodes of this section are selected so as to highlight the students' interactions at the Main Phase of the experimentations and describe: 1) their discussions within their Subgroup as they explored the idea that less variables than the ones that appeared on the Logo program were needed so as to close the "Twisted Rectangle" and 2) their discussion with the other Subgroup in LASAD around this same issue.

We draw our attention, however, on how the students' mathematical activity was fuelled by these discussions and specifically by: a) the fact that they needed to articulate their own ideas in LASAD and explain them to the other Subgroup as clear as possible and b) the fact that they are receiving an idea from the other Subgroup which they needed to try out and decide on its feasibility and usefulness in the process of closing the "Twisted Rectangle". In these instantiations, we also look for manifestation of L2L2 elements

Subgroup's B idea: One of the values is redundant

The students of Subgroup B, in their attempt to close the Twisted Rectangle, they manipulate the three sliders of the Variation Tool. Through this action, they attribute each time different values to the three variables of Logo program that generates the figure on screen.

Figure 3: Attempting to close the figure for f=126.5, w=193 and z=78

As multiple times they have almost closed the figure, but haven't really managed to do so yet (Figure 3), the students come up with the idea that *one of the values is probably not needed for closing the figure* and thus they should manipulate only the sliders corresponding the other two. As they believe this could bring the Group closer to achieving the common goal, they share this idea with Subgroup A through their LASAD discussion map.

S3: I believe that one should go…. It's….. how do we say that? Redundant?

S4: Redundant… We may remove the one line [refers to the slider's numberline]

S3: [Types in LASAD]…have just two values, because may be the one is redundant and makes the shape becoming larger. A suggestion.

Having entered their contribution in LASAD (Contribution no 2 – Figure 3), the students of Subgroup B move back to the microworld in an attempt to solidify the idea of "removing one of the values". Being quite focused on the manipulation of the sliders for closing the figure, the students find it easier to eliminate the effect the third value has on their figure, by simply placing the third slider's pointer on the zero value. Their explorations from this point on, move to a more specific level as for what needs to be done ("have a value equal to 0", instead of "removing it") and share the results of these explorations with Subgroup A by entering a "Claim" in their discussion map (Contribution no 4 – Figure 3).

S3: [manipulates one slider at the time] This one should go. Because with this one you can do that and with this one you can make it come closer. I believe this one is redundant.

R2: So you say this is redundant. How should we make sure? What should we do?

S3: Let's make it 0.[change slider value]

S4: Now it is almost closed

S3: Let's go to the camera. Ahhhh!!!! [Laughters – it's not closed] I know what we'll do

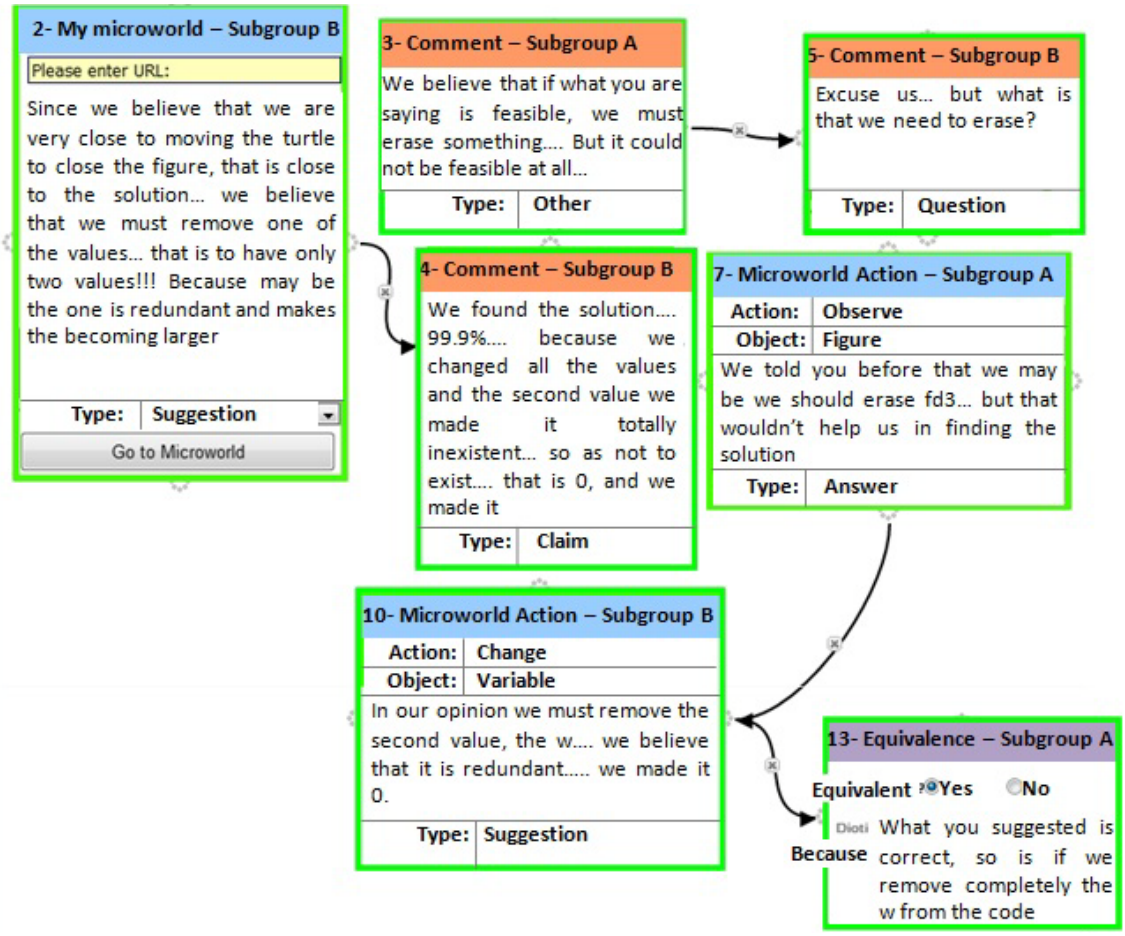


Figure 4: The LASAD map Subgroup A and B use to discuss their ideas on how to close the shape (translated in English)

Subgroup's A idea: Erasing a variable

After reading Subgroup's B contribution about "removing one of the values" (Contribution no 2), the students of Subgroup A move to their microworld to explore if this idea is feasible. However, it seems that while trying to give an answer to Subgroup's B suggestion, they come up with another idea as for how to make two instead of three variables have an effect on the figure they are trying to close.

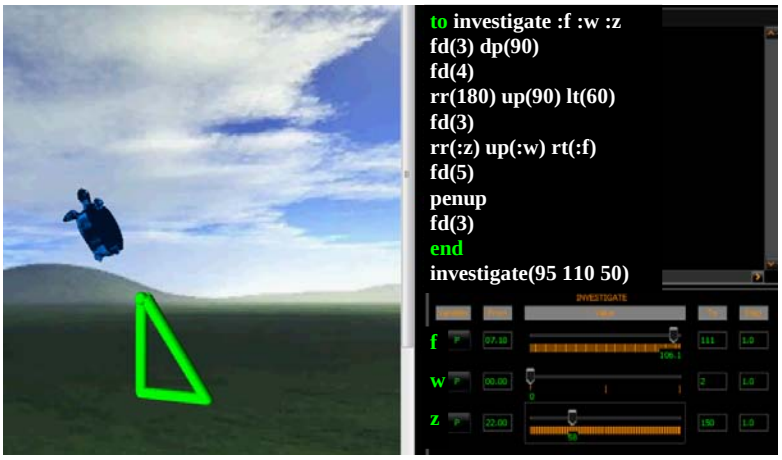
R3: What do you suggest to do?
S1: Let's try to erase something from here…[the Logo code]
R3: What do you suggest to erase?
S1: A letter to start with… a letter… one of the commands.[they explore which variable/command controls which turtle movement]
S2: Just tell them what is that WE believe and then we will try answering them. But I still don't know what answer to give them…

These students are less focused on the way the manipulation of the sliders affects the figure and pay more attention to the Logo program and to the way each Logo command corresponds to specific turtle's moves and turns that construct the 3d figure. Thus, they perceive the "removing one of the values" strategy proposed from Subgroup B as one to be implemented in the Logo Editor and interpret it as an

"erasing something" action (Contribution no 3 – Figure 4). As the Logo command for the Twisted Rectangle's side that the students attempt to attach to the rest of the shape contains variables instead of constant values, the students of Subgroup A go further with their assumption and explain the need to "remove a value" as a need to erase a "letter" (a variable) from the Logo program or a whole command that encompasses a variable. However, as they don't feel confident about their answer, they choose not to expose this new idea to its full extend to Subgroup B. As a result, the students of Subgroup B, post one more contribution (Contribution no 5) demanding from the Subgroup A students to explain what needs to be erased.

Subgroup's B idea revisited: Make one variable's value equal to zero

Just few moments before that, the same students posted the results of their explorations as they had already revisited their initial idea of one value being redundant and suggest making one value equal to zero (Contribution no 4). However, they omit in their Contribution that the variable to which they gave the zero value was the "w" variable. Realizing that the reason for the misunderstanding is the fact that they hadn't efficiently explained to others how to implement their idea, they insert a "Microworld action" contribution (Contribution no 10 – Figure 4) that offers more details as for which "Action" to be carried out ("change") and for which microworld object needs to be manipulated ("variable").



Subgroup A: Evaluating both ideas

The students of Subgroup A, coming to view the symbolic representation (Logo program), the dynamic Variation Tool and the figure graphically generated on screen as three interconnected representations, validate the idea Subgroup B offers as an equivalent to their own and insert Contribution no 13.

S1: "In our opinion we need to remove a variable"…. Ahhhh….. we agree!!!!!
S2: The "w"? [the "w" variable]
S1: Yes the "w", yes… because we also said that it should be the "w"…
S2: Yes! Because it was the "w" that just rolled the turtle!!!
S1: Yes… I'll tell them that we also found that it's "w".
S2: "f" and "z" are more important. They make it [the turtle] go up, right and left
S1: So should we tell them that we agree?
S2: It's the same if we totally remove "w" or we make it zero.

DISCUSSION

The students of Subgroup A and Subgroup B were both given the Twisted Rectangle

half-baked microworld and were asked to make the figure a closed instead of an open one. The two Subgroups worked independently with the microworld but shared an on-line mutual workspace in LASAD in which they discussed their ideas on how to achieve their common goal as one Group. Taking a close look at the students' activity, we tried to identify how meaning generation processes while working with the half-baked microworld where shaped by their social activity as they discussed both within their Subgroup and with the other Subgroup on how to make the figure close. Furthermore, looking at the students' moves between their on-line discussions and their microworld, we sought to identify specific L2L2 elements that may have influenced the students' meaning generation processes.

Our findings indicate that the meaning generation processes were fuelled by the dialogue carried on between the two Subgroups in the LASAD tool. This dialogue was sustained by the fact that the students constantly moved between their discussion map and their microworld, trying out ideas and making them objects of discussion and reflection both for themselves and for the students of the other Subgroup in LASAD. Each time a new idea towards achieving the common goal (closing the figure) was proposed by a Subgroup, the members of the other one tried it out so as to evaluate and check it for its usability with respect to the Group's goal.

At the same time, the members of the Subgroup initially suggesting the idea, revisited it and come to reflect on it so as to make it more explicit for the others (which demanded explanations if they felt that they didn't understand the details), extending the original idea and offering new insights on how to implement it . Reflecting on both approaches (their own and the one offered by the students of the other Subgroup), the two Subgroups came to put all ideas suggested not only under peer assessment processes but also under self-assessment processes. Moreover, Subgroup A, being proactive, used the feedback and experience from the other Subgroup's explorations and taking control of their understanding as a Group, decided that the two ideas were equivalent (Contribution no 13).

Meaning generation, in this case, was also fuelled by the fact that the students evaluated and monitored the progress they made as a Group towards the common goal. They assessed specific learning outcomes as important for the Group's understanding and re-organised accordingly their activities. All these L2L2 elements appear to play a specific role in students' further explorations with 3d Math and discussions in LASAD.

The outcomes of this small-scale pilot study were used to design the main study which was implemented with the participation of 10 9^{th} grade students for 26 school hours.

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