A PROBLEM-SOLVING EXPERIMENT WITH TI-NSPIRE

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This paper focuses on a problem-solving experiment which was an important part of a study of how students and teachers use laptops in their classes with TI-Nspire CAS technology and software, with or without concomitant use of handheld devices. Of particular interest has been the use of this technology for improving students' mathematical learning, problem-solving methods and deeper understanding of mathematics. Eight classes of students in theoretical programmes at upper secondary level in Sweden had continuous access to TI-Nspire CAS in mathematics during a whole semester. They used the software, and in some classes handhelds, during a whole course and also implemented the national test for the course on their laptops.

Key words: CAS, classroom practice, problem-solving, TI-Nspire, upper secondary

INTRODUCTION

Calculators as well as computer software have been used for some time in mathematics classrooms. Calculators have changed over the years, from basic calculators to graphing ones, and now advanced calculators working with computer algebra systems (CAS) that include dynamic graphs and geometry. Simultaneously, computers have changed from being large and rather rare in mathematics education into smaller, mobile units (laptops) that can more easily be used in instruction. At the same time, software has changed from mathematics programs for specific tasks to ones which are much more flexible. One such software for laptops is TI-Nspire, which can be used with or without CAS. It also can be combined with handheld units in a teaching and learning system and thus is possible to use in a variety of ways.

The data for this paper comes from a larger study of how students and teachers from 8 classes used TI-Nspire technology in classroom work in a regular mathematics course at upper secondary level. In total, 133 students and 11 teachers participated in the larger study. Two of the classes took their first course (Ma A) and six of them their second course (Ma B). The mathematics in both these courses included different aspects of algebra and functions. In this paper, I focus on the results of one of the methods used in the study; a problem-solving experiment in which the students were given a longer task with several parts, of which the use of algebra and functions was central. Some examples of students' solutions and reflections after working with the tasks are presented, along with a discussion of the implications of this experiment. The complete report of the whole study is given in Persson (2011).

THEORETICAL FRAMEWORK

Balling (2003) distinguishes between the use of software and calculators as *calculating tools*, *teaching tools* and *learning tools*. When they are used mainly for facilitating calculations, they function as calculating tools. When the teacher takes advantage of their possibilities to illustrate and show important features of concepts and methods, they are used as teaching tools. Finally, when students use them for exploring mathematical objects, to discover concept features and to solve problems, they have the role of learning tools.

A tool can develop into a useful *instrument* in a learning process called instrumental genesis (Guin & Trouche, 1999), which has two closely interconnected components; *instrumentalization,* directed toward the artefact, and *instrumentation*, directed toward the subject, the student. To utilise the affordances of these processes requires time and effort from the user. He/she must develop skills for recognizing the tasks in which the instrument can be used and then must perform these tasks with the tool. For this, the user must develop instrumented action schemes that consist of a technical part and a mental part (Guin & Trouche, 1999). In the present research project, TI-Nspire CAS calculators together with the emulating computer software constitutes the physical parts of the instrumentation process.

The term *resources* is used to emphasize the variety of artefacts that can be considered: a textbook, a piece of software, a student's worksheet, a discussion, etc. (Gueudet & Trouche, 2009). A resource is never isolated; it belongs to a set of resources. When the process of genesis takes place, a document is produced. The teacher and the students build schemes about the utilization of a set of resources for the same class of situations, across a variety of contexts. This process is called *documentational genesis* and also demands time and effort from the users (Gueudet & Trouche, 2009).

The TI-Nspire environment has been studied for example by Artigue and Bardini (2009). They list why this type of technology is considered novel and special, including: its nature; its file organizing and navigation system; its dynamic connection between graphical and geometrical environments and lists/spread sheets; as well as its possibilities to create variables that can be used in any of the pages and applications within an activity. Aldon (2010) has studied the use of TI-Nspire calculators, and assumes that the calculator is both a tool allowing calculation and representation of mathematical objects but also an element of students' and teachers' sets of resources (Gueudet & Trouche, 2009). As a digital resource, these handheld calculators possess the main functions required for documentary production.

Weigand and Bichler (2009) also have researched the use of calculators, and formulated some interesting questions for research, such as:

- When working with new technologies, polarisation occurs in that some students benefit greatly from symbolic calculators use, whereas for other students, SC [Symbolic Calculator] use inhibits performance or even decreases performance. Are there ways to get all students convinced of the benefits of the SC?

- The reasons for non-use of the calculator are on the one hand the uncertainty of students regarding technical handling of the unit and on the other hand a lack of knowledge regarding use of the unit in a way which is appropriate for the particular problem. Is there a correlation between these two aspects?
- The responses of the students confirm that familiarity with the new tool requires a very long process of getting used to it. It is surprising that it took almost a year to establish familiarity with this tool for students to use it in an adequate way. After one year of SC use, confidence in and familiarity with the SC grow. However there is still a large group of students who experience technical difficulties when operating the SC. Will there be ways to shorten this period of adjustment? (pp. 1199-1200)

AIMS OF THE STUDY

The larger study considered the teachers' and students' perspectives as well as the cognitive and affective outcomes. In particular, the ways this technology was used according to Balling's (2003) classification and the levels of students' development in the instrumentalisation process were the focus. The two specific questions addressed in this paper are:

1. What skills in using TI-Nspire technology for problem-solving and in exploring mathematical tasks do the students show after working with it for a significant period of time?
2. What examples can be found of how the instrumental and the documentational geneses have developed during the project?

The first question was partly researched through the problem-solving experiment which is described here, and the second through comparing the results with what the students were able to perform at the beginning of the study.

Interviews with students (2 from each class) had been made near the start of the study, and the end of the study both students and teachers were given separate questionnaires. Directly after the problem-solving experiment, focus groups of students were interviewed about what they had experienced working with the different tasks. Some of their responses will be presented below along with the results from their solutions to the problems.

THE PROBLEM-SOLVING TASKS

The students were presented with problems that were constructed on three levels: The first level involved the students doing ordinary calculations and/or readings graphs. The next level involved some more complicated calculations that required the students to compare different answers and make decisions. At the final level, an exploratory task was given to the students who had to write their answers in plain text. The intention was to create tasks that were close to Balling's (2003) three types of technology use – calculating tools, teaching tools and learning tools.

The problem provided to Ma A classes was solved by the two classes taking their first course (27 students). It was called “Holiday cabin” and described three holiday companies with different fee policies:

- ”Stugbyar AB” takes a basic fee of 1250 kr and then 100 kr more per day.

- ”Semestersol” takes no basic fee, but has higher cost per day, 250 kr.

- ”Strandängen” has a fee per week, no matter how many days you stay during the week. The price for the first week is 1500 kr, but you can stay another week for 950 kr, and after that 950 kr for each week or part thereof.

The students first calculated the fee for specific time-periods (3 days, a whole week and 10 days) and determined which alternative was the most favourable for each period of time. Then they were asked to represent the fee policies with functions and graphs, and finally they had to sort out which company had the smallest fee for all possible time-periods. They were also asked to explain why it is difficult to represent the fee policy of "Strandängen" with a function. A special interest in the study was to observe how the students handled this within the platform. This type of function, a "staircase"-function, was not specifically addressed in the syllables for the course. In the final task, the students' ability to make an overview of a rather complicated problem and to communicate this was tested.

The problem for Ma B was solved by six classes (96 students). It was called “Intersection points” and was based on two functions, one quadratic and one linear that intersected ($f_1(x) = x^2 + 1$ and $f_2(x) = 2x + 4$). These were initially provided through the Graph application. First the students were asked to read and note the points of intersection; then the constant term of the linear function was altered so that there was no intersection and they had to comment on what happened. Secondly, the students had to find out what the constant term in the linear function (instead of 4) was in order to get two, one or no intersections (see figure 1). Thirdly, they were asked to solve a non-linear system of equations that exactly reflected the graphs in the first part (the students were supposed to discover this). Finally a parameter *m* was introduced in the linear function for the constant term, and they were asked to solve the system again and explained why this general solution created two, one or no solutions for the system:

$$\begin{cases} x^2 - y = -1 \\ 2x - y = -m \end{cases}, \text{ with the solutions } \begin{cases} x = -\sqrt{m} + 1 \\ y = m - 2\sqrt{m} + 2 \end{cases} \text{ or } \begin{cases} x = \sqrt{m} + 1 \\ y = m + 2\sqrt{m} + 2 \end{cases}$$

The students were asked to reflect on the two general solutions and explain why these created different types of solutions for varying values of *m*. Using parameters in equation solving and explaining the outcomes is not a part of the syllables for the course, and a special interest in the study was to see how the students would interpret this new experience.

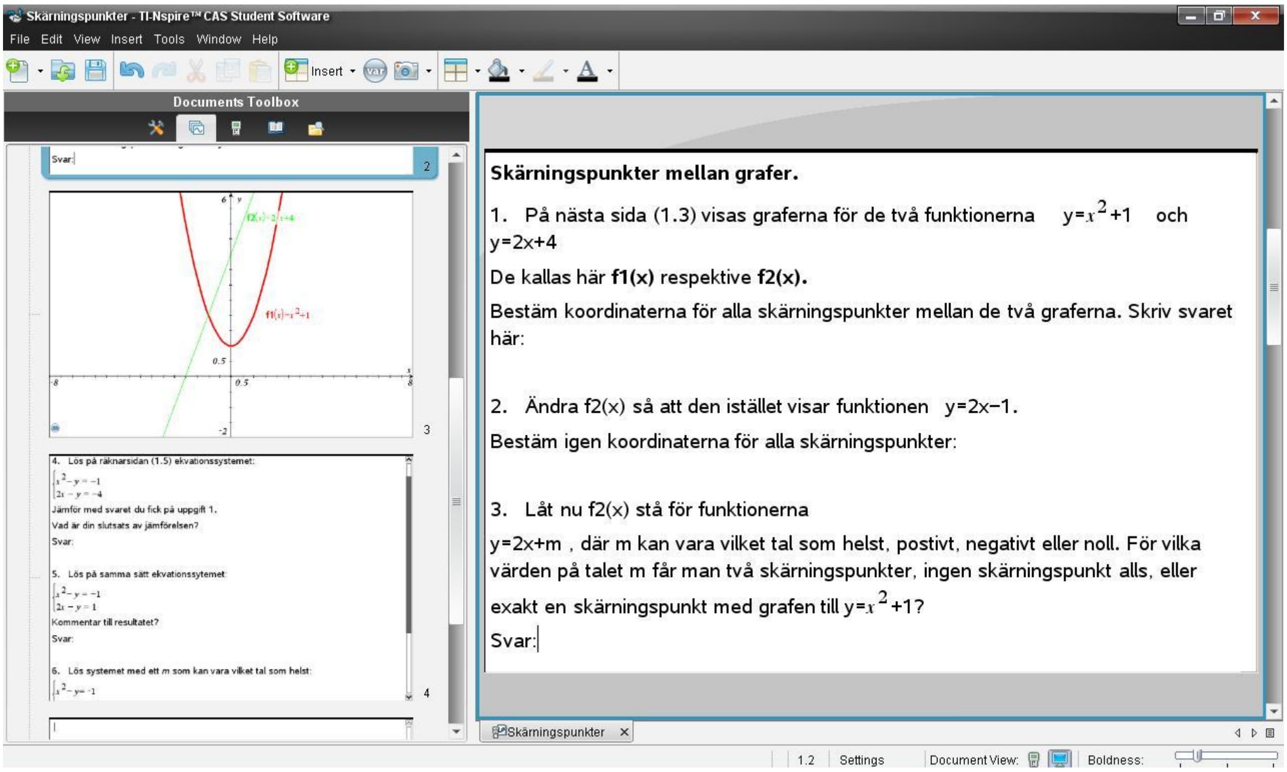


Fig 1: The first part of the problem for the Ma B students, concerning intersection points between graphs.

The problems, with all the instructions, were given to the students by means of the special TI-Nspire-files called tns-files. The students were asked to work individually with the tasks in the classroom. They also had to work within the given file and to present all their solutions in the Notes application within the file. The tns-files were then handed in, identified with the students' names. During the whole experiment the students’ work with the task was observed by the researcher.

SOME RESULTS

The observation of the classes showed that they handled TI-Nspire as a tool in a mainly productive way, both through calculators and on laptops. Some features of the technology were new for them, but in most cases they coped very well with these. The difficulties seemed to lie more with the mathematics that the students had to engage with in the problems, and this was connected to the students' general mathematical abilities. Their problem-solving skills with TI-Nspire were generally good, with only a few exceptions. Many also managed to give good answers to the more difficult parts of the problems. The analysis of the files that were handed in was built on an 'a priori' analysis of the mathematical structure of the problems, and on the anticipated possible ways to solve them. It also included the types of sub-tools that could be used in the problem-solving process, and the representations the students worked with. With these, students' versatility in using the technology as a tool within the three classified types could be established. In the final analysis of each student's file also 'post priori' considerations were used, such as the quality of the students' answers, how they communicated these and in what ways they showed signs of mathematical reflection. Each file was taken through the analysing process several times in order to secure the validity of the results, which are shown in table 1.

	Calculating tool	Teaching tool	Learning tool
Ma A (n = 27)	24	21	10
Ma B (n = 96)	93	83	57
Total (n = 123)	117 (95 %)	104 (85 %)	67 (54 %)

Table 1: Results of the problem-solving experiment, classified according to Balling's (2003) definitions.

The methods used by the students in the solving process varied. Some worked more with algebraic methods, using the CAS application, others more with graphic solutions. For example, some students demonstrated that they can manipulate graphs directly by dragging them up and down. Others instead worked with the equations for the functions and made changes in them, either in the entry line or in the formula inside the diagram. Some Ma B-students were familiar with the way equation systems were entered in CAS and could combine with the Solve-command. Others used more indirect methods. Most of the Ma A-students were quite unfamiliar with broken functions, especially the way these are possible to handle within TI-Nspire. These differences can be explained by the way that the teachers involved had, or rather had not, let the students work with this type of functions and objects in TI-Nspire during the study.

The problem for Ma A did not demand much use of CAS, but the one for Ma B did. Some students had specific difficulties, and asked for help with some of the practical components involved in using it. Some students also thought that the solution they got to the equation system with the parameter was ‘weird’ and therefore probably wrong. Data from the focus group provided further details about this:

Ma A:

Male student 1: It was things that we've done before.

Male student 2: Things that we knew, nothing particularly new.

Interviewer: Was the software well suited for working with such a problem?

Male student 2: I think CAS is great for this kind of task.

Female student: The first two questions were not so difficult, but the third one was pretty tricky. But we have had similar questions.

Interviewer: It seemed as if you were a little inexperienced at reading the intersections of the graphs?

Female/Male students: Mmm. We had not done that so much before. And it was difficult with "Strandängen" [the staircase function].

Ma B:

Female student 1: It was pretty similar to what we usually get in math books.

Female student 2: And sometimes we get ... and we will work in pairs. And you probably get one of those sent to the calculator.

Female student: We have not used this ‘solve’ in systems of equations before, but we have made it algebraically by hand.

Female student: The exercise was not so difficult in itself, but we do not usually do such exercises.

Interviewer: Can you mention something specific that was an obstacle for you?

Female student: Exercise 6, the penultimate. That kind of task we have not had. Like having an m besides x and y.

Nevertheless, most of the students had no substantial difficulties working with CAS. In analysing the solutions, having access to the calculations that the students had made in the Calculator application in the tns-files, gave extra information. Most students had usually not thought of deleting their mistakes, so it was possible to follow their ways to a solution. In figure 2 two examples of students' calculations are shown.

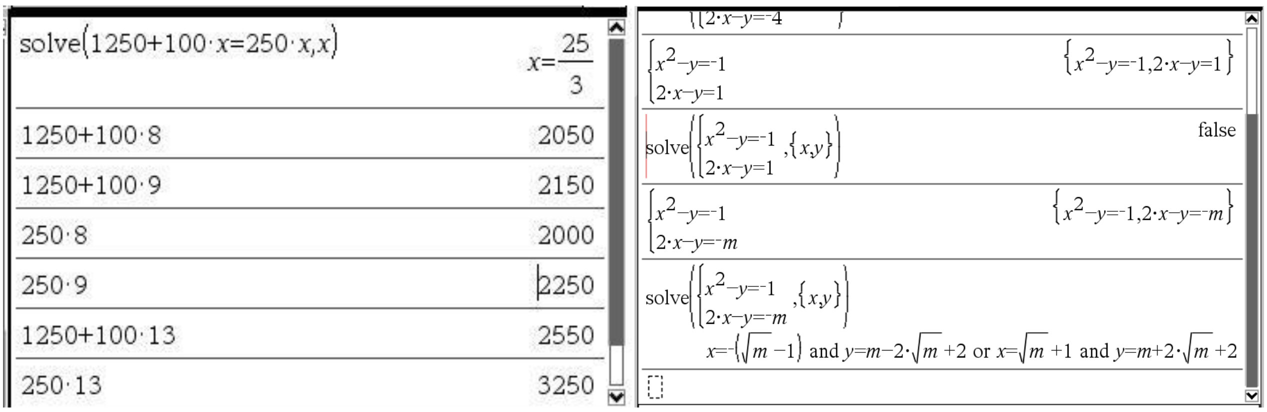
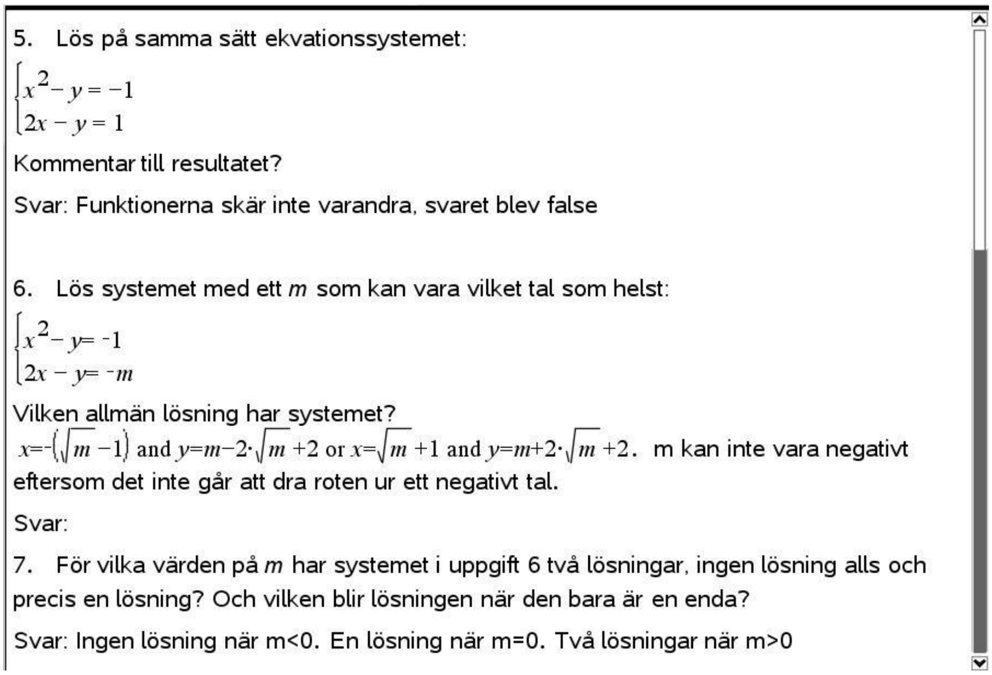


Figure 2: Excerpts from two students’ tns-files (CAS application). The problem for Ma A is given on the left and for Ma B on the right.

Writing text within TI-Nspire seemed to cause problems for many students, especially those with handheld units. They were generally not used to do so within the system. The written answers to the different tasks were with a few exceptions very short, using mostly symbolic mathematical expressions and very few words

(Figure 3). When asked about this, the students explained that it was hard for them to write, because the keyboard was small, it did not contain Swedish letters like å, ä and ö (they had not discovered that they easily could access these letters in ways similar to how they used their cell phones). There was, however, one exception. In one of the Ma A-classes, the method of distributing tasks through tns-files and for the students to hand in their solutions using written text was a common daily procedure. For this class, which used laptops, writing text represented no obstacles.


5. Lös på samma sätt ekvationssystemet:

$$\begin{cases} x^2 - y = -1 \\ 2x - y = 1 \end{cases}$$

Kommentar till resultatet?

Svar: Funktionerna skär inte varandra, svaret blev false

6. Lös systemet med ett m som kan vara vilket tal som helst:

$$\begin{cases} x^2 - y = -1 \\ 2x - y = -m \end{cases}$$

Vilken allmän lösning har systemet?

$x=-(\sqrt{m}-1)$ and $y=m-2\cdot\sqrt{m}+2$ or $x=\sqrt{m}+1$ and $y=m+2\cdot\sqrt{m}+2$. m kan inte vara negativt eftersom det inte går att dra roten ur ett negativt tal.

Svar:

7. För vilka värden på m har systemet i uppgift 6 två lösningar, ingen lösning alls och precis en lösning? Och vilken blir lösningen när den bara är en enda?

Svar: Ingen lösning när m<0. En lösning när m=0. Två lösningar när m>0

Figure 3: Example of from a Ma B student's solution with a rather short text. The translated answers to the questions are:

5. ***The functions do not intersect; the answer was "false".***
6. ***$x=-(\sqrt{m}-1)$ and $y=m-2\cdot\sqrt{m}+2$ or $x=\sqrt{m}+1$ and $y=m+2\cdot\sqrt{m}+2$.***
   m cannot be negative since you cannot take the square root of a negative number.
7. ***No solution when m<0. One solution when m=0. Two solutions when m>0.***

Laptops are normally not allowed when students sit the Swedish national tests, something that the students in the project had to do at least one. So special permission to use laptops for research purposes was applied for at the Swedish National Agency for Education. Permission was granted on two main conditions: First, any communication between students or through the Internet was forbidden, and second, unwanted files that could be used for cheating should not be accessible. Only the software TI-Nspire was allowed for the students to use. These conditions were met by the teachers, and laptops were successfully used during the tests at the end of each of the courses, Ma A and Ma B. This showed that it is possible for students to complete the national tests using laptops. If this is implemented at a larger scale in the Swedish school system, solutions like turning off the Internet during the test or creating special 'test clients' for the laptops with USB memories will be possible.

DISCUSSION AND CONCLUSION

Several students had in their first interview explained how difficult and complicated TI-Nspire seemed the first time they used it. It contained so many ‘things’ that they hardly knew where to begin. However, most of the students also answered that after a short period of time, when they had become familiar with the software and/or the handhelds, it did not seem so complicated. During the project, the students’ versatility in using the technology had progressed substantially. Many of the difficulties they saw in the beginning disappeared, although even at the end of the study there still were a few students who had significant problems with the use of the software or the handhelds. In the teacher’s questionnaire 9 out of the 11 teachers answered that some students continued to have difficulties with the use of TI-Nspire, although most students had made progress in their ways of working with it. Five teachers also answered that some students enjoyed exploring TI-Nspire in order to find new functions, and they often shared what they found with other students and sometimes also with the teacher.

> Teacher: But then there are some students who understand a little quicker and can show the others. So suddenly you have a whole staff that is helping. And it's good for you to have that.

The ways in which students documented their work with tasks and problems showed very little progress during the project. To some degree this was due to the fact the teachers rarely used the possibilities to work with files with many pages or pictures, and that the students did not use the Notes application in TI-Nspire to really document their work or to hand in solutions to tasks, with the exception of one class mentioned above, for which it was a fairly common procedure.

The problem-solving experiment, and the reflections made by the students in the focus groups afterward, showed that the students could use the technology as a tool in their classroom work, but also that there were possibilities for taking these uses further. If the students had been given longer problem-solving tasks to work with, they could have been able to solve even more difficult problems. They could also have become more used to writing longer answers to the tasks, where they could give more explicit arguments for why their solutions were valid and perhaps also present proofs. In the new Swedish curriculum, two *abilities* are especially emphasised; ability to make argumentations and proofs, and ability to communicate. This type of technology and software could be of assistance in the learning process.

Students in the study showed significant progress in the instrumental genesis and also to some extent the documentational one. Nevertheless, a much more complicated process is required, and the results suggest that this may take a long time, maybe several years. It is difficult to insert technology as an organic part of the resources of a "document" (Guedet & Trouche, 2009) which represent whole work sessions or lessons in mathematics. However, even here a certain development was observed, and there were signs of a continuation of the process involving the TI-Nspire for both teachers and students, at a higher level.

Perhaps the most important result of this study is how TI-Nspire has been used in regular education in upper secondary courses. The various possibilities (Artigue & Bardini, 2009), of a technical, mathematical and conceptual nature, had the opportunity to appear in this longitudinal study taken over several months. It was interesting to be informed about the students' experiences and opinions of the technology. The technology was seen as contributing to speed and accuracy as well as providing a variety of forms of representation. It is also obvious that CAS represents a difficulty, especially for low-performing students, and simultaneously has an incredibly powerful potential in mathematics education. Only a few students are able to take full advantage of this potential, even though most of them can use CAS in a satisfactory way. Experiences from the use in the national tests were also positive. The barriers that existed for the use of laptops could be effectively eliminated, and this shows that it is possible to perform one of the sections of each national test with laptops as aids.

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