WHY AND HOW TO INTRODUCE NUMBERS UNITS IN 1ST-AND 2ND-GRADES

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Abstract. Learning and teaching the decimal number system is well known by researchers as more difficult than the common adult user thinks of, This paper aims at studying some semiotic problems -how are denoted the quantities and what quantity denote the numbers- from different points of view, epistemological, institutional, didactical. It develops a few propositions for solving them, based on the introduction of a language referring explicitly to ten (numbers units) and specific materials (hand fingers) giving meaning to this language.

INTRODUCTION

According to our function as teachers educator we were asked by the whole team of a school (grades 1 to 5) for helping with the teaching of natural numbers along the five grades. This request was the pretext for pursuing the reflexion about the teaching of the written- in the Hindu-Arabic and spoken-number systems, learning from recent French didactic research, notably through institutional (Chevallard 1992) aspects.

This paper focuses on the teaching and learning of the spoken- and written-multidigit-numbers by 6-to-8-year-old students (in France, grades 1 and 2). In our paper, "26" is called a *written-number* whereas "twenty-six" is called a *spoken-number* and “2 tens 6 ones” a *numbers-units-number*. For young children learning and working with numbers is far from easy. Among the knowledge they have to learn and to understand: how the multidigit-numbers are constructed from a semiotic point of view? How the written-multidigit-numbers and the spoken-numbers denote a determined quantity? We will address the question of teaching spoken- and written-numbers between 1 and beyond 100 from a semiotic, didactic and institutional point of view.

Our main question is: what tasks should contain a guide in order to engage grades-1st-and-2nd-teachers to revamp their teaching of the written- and spoken-decimal systems of numbers?

SOME ELEMENTS ABOUT THE WRITTEN AND SPOKEN NUMBERS

Written numbers: semantic and syntactic aspects

We consider the multidigit-numbers like a language with its semantic and syntactic aspects. Why this point of view? The syntax seems simple, but powerful: any digit-concatenation denote a numeral and a quantity. Each digit of the ordered list 1 to 9 denote a quantity, 1 for one objet, and the successor, one more than the precedent.

For a quantity bigger than 9, 1 moves to the left and a digit 0 takes the right place, following by 11 12 13...: the going on written-numbers list respects again the same progression for the right digit, up to 9; then the digit on the left becomes 2 and the right one follows again the progression from 0 to 9, and so on. It is what we named the recursive aspect of the written-numbers list. A good understanding of written-numbers recursive algorithm permits to order written numbers without understanding the semantic aspects.

With regard to semantic aspect, how to know what quantity denote a multidigit-number? "Written numbers as partially iconic signs" (Ejersbo & Misfeldt, 2011, p.301), we will precise only very little iconic: number of different lengths can be easily compared. But how to understand that 52 is different and major than 25 and why? The place value system owns genius aspects, but these aspects remain not known by young students who often consider 52 equal 25 is another way to denote 7.

The specific role played by the digit 0 in the written numbers is a semantic aspect: 0 indicates the lack of an isolated power of ten, e.g. 0 indicates no isolated one in 450, no isolated ten in 203, still there are 450 ones in 450 and 20 tens in 203.

A few words about spoken-numbers

In the French language, the names of the numbers units are different from these of the powers of ten, as summarized in the table below: with units, 423 is 4 "centaines" (4 hundreds) 2 "dizaines" (2 tens) 3 "unités" (3 ones) but with the names of the powers of ten, $4 \times 100 + 2 \times 10 + 3$ is 4 times "cent" (hundred) plus 2 times "dix" (ten) plus 3. Another difficulty: French language uses the same word, "unité" in a context of units of measure or account, but also to describe the quantity "one".

	1	10	100	1000
English	one the ones	ten the tens	hundred the hundreds	thousand the thousands
French	un, une les unités	dix les dizaines	cent les centaines	mille les milliers

Table 1: Names for numbers and for numbers units in English and French

Mounier (2012) highlights four aspects of the spoken-numbers: "forty six" is 1) four times tens plus six: multiplicative, 2) forty plus six: additive, 3) a place in an ordinate list of words, one, two…, forty five, forty six: ordinal, 4) a place in a list of words (ten, twenty, thirty…) with counting words (one, two, three…, nine), six after forty: ordinal with marks.

NUMBERS UNITS: A DIDACTICAL NECESSITY

To tackle with the signification of spoken- and written-numbers, we stress the introduction of another system of denotation of quantities, using "numbers units" (Chambris 2008): ones, tens, hundreds, etc. - which are units of account - naming by specific words what many teachers in France and some other countries (Ma 1999, Thanheiser 2009) only see as the names of the places of the digits.

The systematic use of numbers units is a way to make precise the decimal systems of numbers. Freudenthal (1983, p. 90) spoke of "decimal bundling" and "positional arrangement of the bundles", but like Fuson & al (1997), Chambris (2012) emphasizes the necessary and difficult double point of view on the bundling: 10 ones is 1 ten, 1 ten must be understandable as a multiplicity (10) and a whole (1 unit of ten), that Thanheiser (2009 p.253) names "a flexible view of unit types (e.g. ten)". This is an important stage aiming for spoken and written number systems understanding, a part of numbers "profound understanding" (Ma 1999 p. 122).

Why to introduce these words? We synthesize some arguments of Chambris (2008), partially used in Van de Walle (2007). First, they enable teachers to reveal and students to approach the coherent iterative organisation concatenating digits and to estimate the "order of magnitude" of the number. Second, they were used in pedagogic arithmetic theory to describe numerations -for example Condorcet 1794- and in ancient text books. Third, they permit to express the systemic relations between the different places of a digit in a number: 1 ten = 10 ones = 10; 10 tens = 1 hundred = 100 ones = 100; 10 hundreds = 1 thousand = 100 tens = 1000 also 1 ten of 10 tens... Naturally their use is bounded to the understanding of each term, to the evocation of the quantity denoted by each numbers unit: the use of numbers units needs the introduction of activities that help students construct the sense of these units. Fourth, they highlight many useful relations for computation: 24+58 is first 7 tens and 12 ones, and thus 82 (this explains the carries in the column operation); for the integer division of 245 by 7: the decimal aspect takes into account that 245 is also 24 tens and 5 ones, 245 doesn't contain only 4 tens, but 24 tens, and thus the quotient is 3 tens.. and 5 ones. Fifth in the later teaching they will prepare and strengthen the SI units (length, weight.....). There is a sixth reason to introduce the numbers units: the multiplicative aspect of the spoken-numbers can be interpreted with the numbers units: forty six is four tens and six ones, it may help to switch from an ordinal to a multiplicative aspect.

Numbers units own the property to be said and written exactly the same way whereas written-numbers and spoken-numbers are often no congruent in French like in other languages (Ejersbo & Misfeldt, 2011), that is a big problem for teachers when speaking about quantities (Mounier 2010). Numbers units may be helpful for the teachers to *speak* the quantities. **Numbers units bridge a gap between spoken- and written-numbers** as a flexible "*base-ten language"* (Van de Walle 2007 p189): standard oral name *forty two*; base-ten oral name *4 tens and 2* **;** standard written *42*; explicit base-ten written *4 tens and 2 ones*. *4 tens and 2 ones is also 3 tens and 12 ones*, etc. This conversion IN the base-ten-language is formed and proved by two organisations of the same collection (two partitions Ross 1989). This is a seventh reason to teach numbers units and it has something to do with the third reason.

THE FRENCH INSTITUTIONAL STATE OF MULTI-DIGIT NUMBERS TEACHING

For US researchers and many teachers (for example the readers of Van de Walle 2007) numbers units and their consequences may be very usual. But in France, as we will partially see, it is not: the contexts of national curricula (contents and developments of theses contents), usual teaching and training practises, type of national dominant researches... can be very different. So we now present how multidigit-numbers are taught at French primary school.

At pre-school, children are used to seeing and reading the written-numbers (usually from 1 to 30) like they are used to the written-language. From grade 1, they have to enter in the signification of codes already there, the multidigit-numbers. The French curriculum texts don't give any indications about natural numbers teaching complexity: in the most recent ones (2008) "place value" or "base-ten-number system" is not written anywhere. For example, what concerns expectations:

- 1^{rst}grade-students should "*know (write and name) natural numbers less than 100; produce and recognize additive decompositions of numbers less than 20 (addition table); compare, order, frame numbers less than 100; write numbers in ascending or descending order ; know numbers-less-than-10 doubles and numbers-less-than-20 halves*".
- 2ndgrade-students should "*know (write and name) natural numbers less than 1000; place and locate these numbers on a number line, compare them, order them, frame them; write or tell sequences of numbers 10 by 10, 100 by 100, etc; know usual-numbers doubles and halves*". (MEN 2008 p2)

Further, Chambris (2008), using the anthropological theory of didactics (Chevallard 1992), shows that in French primary school, from the 1980's, numbers-units have "disappeared". They remain as "names places". As "units", they have been replaced by powers-of-ten, written in figures (1, 10, 100, 1000). Teachers, neither most teacher educators, are not used at all into teaching units when they teach whole numbers. For instance, 2x100+3x10 (or even 200+30) has replaced 2 hundreds 3 tens but 3 is still called the tens digit in 230. We don't know what is the institutional state in other countries but there are clues that similar things may have happened at other places: Fuson (1992) wondered why students wrote such writings and indicated that these types of documents do not help to learn the number words; Ma (1999), Thanheiser (2009) let us think that numbers-units may be taught as places, and not as units, in the USA.

Recent French didactic research highlights the weak knowledge of the in-service teachers. It may be a possible consequence of the curricular text that don't focus on these points: the students are taught the positional aspect of the written numeration, 52 is different from 25, they don't have the same position in the usual well-ordered number list 1, 2, 3..., 25..., 50, 51, 52 ; but the teachers don't systematically emphasize the decimal aspect nor the quantitative reason of the order of written numbers: 52 denotes 5 tens and 2 ones while 25 denotes 2 tens and 5 ones. In this form and with two-digit numbers, it looks like only a vocabulary question and is often taught like that. Even in greater numbers, many teachers (especially in the studied school) reduce the tens in a number to the isolated tens, highlighting the tens digit. That teaching can create a didactical obstacle (Brousseau 1997) preventing the students from constructing and/or understanding computation algorithms. When the teachers intend to teach the decimal aspects of the written-numbers, they use the spoken numbers, especially the known order of the multiples of ten: ten, twenty, thirty... (Mounier 2010).

The question is now: how to provide teachers some tasks to teach 1st-and-2nd-grades students numbers-units in relation with the written-numbers and quantities so that they better understand the semantic of written multidigit-numbers? First we must choose materials, easy to find and to manage, giving sense to numbers units. Second we must convince them of the usefulness of the introduction of the numbers units.

HOW TO CHOOSE MATERIALS ?

We agree with many "*base-ten ideas*" developed in Van de Walle 2007 (chapter 12) : counting plays a key role, particularly the dialectic between counting by ones, counting by groups (of tens objects) and singles, counting by tens and ones.

Unlike Van de Walle (2007, p193) we think we don't let the students choose their way of grouping, we have to impose grouping by ten, even if grouping by two or five may be more spontaneous. Certainly grouping by five and counting orally (by five or by ten...) could be interesting and it is easier to control 5 bundles than 10. But there is a bigger semiotic distance between numbers of fives and decimal standard spoken- and written-numbers: 43 and forty-three is 8 fives and 3 singles and also 4 tens 3 ones which looks more like 43.

Guitel (1975, p.25) stressed the predominance of usual bases in written numerations: 10, 20, 60 as principal basis; 5, 100 and even 10000 as secondary one. From an historical point of view this common characteristic of the mankind offers a credible hypothesis for the choice of ten in many base-numerations over the world and the times. It is a track to analyse how numeration "is bodily- grounded that is, embodied within a shared biological and physical context" (Nuñez & al 1999 p.46).

The English language gives another track: the word *digit* literally means finger or toe too. In English there are two words: *numeral* in a common sense and *number* in a math sense. Numeral can be synonym of digit too.

Counting activities will be in the heart of our didactical proposition in 1st-and-2nd-grades. Our semiotic approach makes us vigilant on the first material we offer to students, it must integrate and naturalize so far as possible the grouping by tens : we choose the hands fingers.

Many school practitioners suggested and suggest their students to use their fingers in order to denote quantities less than 10 or to construct the different additive decompositions of 10. Ladel & Kortenkamp (2011, p.1795) wrote:

> Amongst others, the advantages of fingers and hands are their permanent availability and their natural structure in 10 fingers per child with 5 fingers per hand. The 10 fingers qualify the hands to work out questions about the decimal number system, e.g. "*How many children do we need to see 30 fingers all at once?*" The 3 children stand for 3 tens.

From a didactic point of view, thanks to the human body, the ten looks like a whole (one student) and a multiplicity (tens fingers).

Seeing ten (or several tens) as the set of my hand-fingers (or a reunion of several-persons-fingers) would contribute (as mental picture and associated properties and processes) to create "a concrete internal mental image" (Thomas & al. 2002) of the ten unit with its double meaning: a one is an unit of account, but a ten (the ten hand-digits of a person) too, 2 tens is not 2, it is bigger than 2, it is twice ten. It is not easy to exceed 100 fingers, but the amount of one hundred will have a strong image, 10 students raise their fingers, as many students as fingers of the hands of a student Using special expressions (numbers units), writing relations of one unit to another in the context of fingers and of numbers, expanding formally these relations contribute to reinforce the conceptual understanding of written-numbers.

Why to choose the fingers instead of Dienes blocks, for instance? Chandler & Kamii (2009) compare the different statements in the literature: it emphasizes two options. Kamii (2000, 2004) "does not recommand the use of coins, base- 10 blocks or Unifix cubes for teaching tens and ones, reasoning that children need to abstract units of ten out the ones they have in their heads, rather than from objects in the external word." (Chandler et al., 2009, p.97). Others like Van de Walle's (2007, Chapter 12) propose physical models for place value, distinguishing (p191) groupable base-ten models (bundles of sticks, interlocking cubes..) from pre-grouped base-ten models (base-ten blocks,... even money (p.209).

Lamon (1996, p188) shows that, for older students faced to in partitioning tasks (aiming the fractions), decomposition of a given all into small units appears quite immediately but unitization into composite pieces seems more complex and appears later. The hand fingers permit to work simultaneously two aspects (decomposition and unitization: showing a quantity of fingers like 32 with few children makes visible 32 singles (the fingers) and its decomposition in 3 tens (3 children) and 2; counting a quantity of fingers shown ten by ten very quickly by one person makes necessary the unitization.

Using first the fingers of the students to describe or produce a quantity owns the three properties (Stacey & al, 2001, p.201) of interesting materials for teaching young children: appeal to the students, accessibility, but especially epistemic fidelity (representing order of magnitude, bundling into tens for numbers under 99, constructing of an incorporated semiotic flexibility of the unit). The students will be faced in a second period to other groupable material like wooden craft sticks so that they have opportunities to abstract the unit and transfer the ten fingers model in another context.

A DESIGN PROJECT FOR GRADES 1 & 2

The school asking for help is situated in what is named in France "a sensitive zone". The students of these zones suffer simultaneously bad socioeconomic conditions and weaker evaluation results than other schools despite more State helps. At the moment, these schools are observed by sociologic and didactic researchers, analysing possible origins of the bad learning. We present in this paper the design of a type of activities that respond to certain conditions: short (15 min), repetitive (each day), dynamic, very little expensive in materials and engagement of the students. These activities represent only a part of an entire design about numbers, the part that would consist to embody the concept of unit.

They aim for constructing relation between written-numbers, numbers units and quantities. The usual way the numbers are said in France is not specifically studied in this design. The students are supposed to know (spoken and written) numbers from one to ten at the beginning and also how to realise such sets with their fingers.

Examples of questions that pursue ancient questions with more little numbers:

Type 0: introducing the ten as the first unit that is not a one

The students are asked 1) how many students there are in this group (groups of 5, 7, 11, 15, the entire class room), 2) to show the quantity using their fingers and 3) to write or to say the number if they can. The succession of different numbers will conduct the students to find a way to show more than ten fingers: several students will produce the quantity... The teacher could name and write: *a student has 10 fingers, a ten of fingers. To show with fingers more than 10, more than one person is needed. How many persons with all fingers up corresponds in mathematics with what is named the tens,* e.g. *there is 1 ten of fingers and 5 fingers again, there are 15 fingers, there are fifteen fingers.*

This collective activity is aiming for the construction of the unit, a ten, as a multiplicity (the ten fingers) and a whole: the person - with all fingers up - as a ten, that is universally considered as a difficult knowledge. The following propositions are aiming for pursuing this construction, giving the students more and more individual responsibility in their responses and so, more and more occasion to understand. Two types of collective activities are described: production activities (P), communication (C) ones. In the first ones, it is more the teacher that validates the responses; the second ones enlarge the interaction between students so that they validate the responses and argue about the reasons. Type E refer to individual training.

Type P1: production of a set from a written- or spoken- already known number

The students form teams of 4, 5 or 6; the teacher writes a digit-number on board and asks students for showing this quantity of fingers - in the list of numbers students know - for example 8, 20, 26, 32.... The students could count one by one or progressively by more than one. Each time some configurations are observed, a realisation with the less students as possible will be encouraged.
8: 1 student with 8 fingers up (and 2 fingers down)
20: 2 students with all fingers up
26: 2 students with all fingers up, 1 student with 6 fingers up (and 4 fingers down) ...
Progressively the teacher can let the team propose a solution before trying it.

Type P2: producing a way to write a quantity from a set of fingers

The teacher shows fingers and the students are asked for the quantity. The set of fingers is not showed simultaneously, but successively: producing 23 fingers consists on showing ten fingers twice (twice: fists first closed, then fast opened with all fingers up) and again 3 fingers up.

First the teacher shows quantities in the list of known numbers, then quantities of bigger numbers: students can no longer count one by one, or ten by ten (ten, twenty, thirty..), they must quickly count the times when the teacher shows all her fingers raised, therefore they must count the number of tens... and the isolated ones. They are forced to do this by the situation, the constraints of the situation change their way of thinking (Brousseau 1997) and helps them constructing the unit concept.

Type P3: production of a set from a numbers-units-number

Same activities as P1, but a student can produce its set in front of the classmates, showing the tens successively as learned from the teacher in P2. The way to write or speak the number can be either found by students or given by the teacher or be pended.
The teacher may conclude the activity with mathematical words and write a lesson - we let voluntarily many words in French in the table

8	8	*1 student with 8 fingers up*	*8 doigts*	8 unités
20	vingt	*2 students with all fingers up*	*2 dizaines de doigts*	2 dizaines
26	vingt-6	*2 students with all fingers up, 1 student with 6 fingers up*	*2 dizaines de doigts et 6 doigts*	2 dizaines et 6 unités
32	trente-2	*3 students with all fingers up, 1 student with 2 fingers up*	*3 dizaines de doigts et 2 doigts*	3 dizaines et 2 unités
		7 students with all fingers up	*7 dizaines de doigts*	7 dizaines

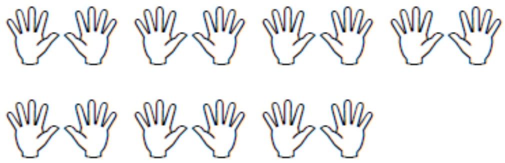
Table 2: What must being remembered

Type C1: communication between students about quantities denotations.

The teacher gives a paper with a number in numbers units to a team, lets students of the team show their fingers (corresponding the quantity) to all the other students who write the response on their slate. The response is then compared to the original that permits validate the communication, comparing same writings (in number-units) or both writings (in number-units and written).

The same can be played by team, two teams A and B are associated, in the first time, team A receives the number and shows the collection of fingers to team B, then the role are reversed , the teacher looks at the students

After collective discussion the number is written with numbers units, drawn fingers and digits.

3 dizaines et 4 unités	*7 dizaines*
3 tens and 4 ones	7 tens
	
34	70
Trente- quatre	Soixante-dix

The name of the number is said and written, with an oral advertising for "oral irregular" names like *soixante-dix* (notice that number names are less irregular in the Belgian French language: *septante -“seventy”- instead of soixante-dix -“sixty-ten”)* :

The chosen numbers are written by the teacher with canonical form (5 tens and 6 ones), non-canonical form (6 ones and 5 tens OR 2 tens and 16 ones and 2 tens) and will engage specificities : no one (6 tens OR 3 tens and 30 ones) or, more simple, no tens (5 ones).

Type E1: Individual exercises with prepared drawn hands labels and persons:

Labels of a person (conventionally supposed with all the fingers up) or labels of hand are available.

1) A numbers-units-number is given, constructing the fingers collection (with person labels or hand labels) and writing the multidigit number are asked the students. Notice that succeeding using "hand labels" reflects a better understanding of the ten unit. 2) A set of drawn fingers is given, writing the quantity with numbers units and with digits are the questions the students must face to. 3) A written-number is given, constructing the fingers collection (with digits or persons labels) and writing it with numbers units are asked the students

It would be progressively important that the teacher proposes too "fingers not totally regrouped in hands".

Type C2: activities of communication but there are more than ten ones, for example 5 tens and 12 ones. The teacher emphasizes the conclusion: 5 tens and 12 ones is also 6 tens and 2 ones. It is usually written 62, then need all the fingers of 6 persons and again 2 fingers up.

Type E2 : Individual exercises

Type C3: activities of communication but there are more than ten tens, for example 13 tens and 7 ones. The number of persons with all fingers up is 13, and again 7 fingers. *Ten persons with all fingers up form ten tens of fingers, in mathematics a hundred of fingers*. *13 tens and 7 ones* is also *1 hundred 3 tens and 7 ones*, usually written 137.

Type E3 : Individual exercises

CONCLUSION

The previous activities have a main goal: to teach the concept of unit -with the instance of ten- betting on the incorporation of the ten as privileged grouping. Using numbers units allows to improve the semantic understanding of 2-digit or even 3-digit numbers; it also allows to speak numbers without using their difficult names. Concerning number names, the core is to memorize the relation between the tens names and how many tens in the tens names: for example four tens is forty, four tens and five is forty five.

The activities are expected to give progressively students more and more responsibility in the control of their responses, integrating the role of tens in written- and spoken-numbers.

We are conscious of the lack of experimental facts that would moderate our development. We would just share our a priori thinking. On the one hand, it is based on some research on learning (base-ten concepts, numbers units as mathematical and pedagogical mean...) and teaching (little trace of numbers-units in French curriculum, little use as place value in usual practices...). On the other hand, it takes into account the constraints we had to face to: convincing the teachers of the utility of a new notion (numbers units), proposing short and regular meaning activities, so that they are not afraid to try them and we can examine in true the effects of their integration.

Nevertheless, some questions rose. How written- and spoken-numbers overlap in the teachers practice? How to manage with the problem of the very irregular number words in French -3 stages from 1 to 100: from ten to 10 to 19 (*dix, onze, douze,... dix neuf*), from 60 to 79 (*soixante* to *soixante-dix-neuf*), from 80 to 99 (*quatre-vingts to quatre-vingt-dix-neuf*).

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