MATHEMATICS TEACHERS' PERCEPTIONS OF QUADRILATERALS AND UNDERSTANDING THE INCLUSION RELATIONS

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Abstract: *The aim of this research is to investigate how mathematics teachers comprehend the properties of specific quadrilaterals, how they construct the relations and classify quadrilaterals. This research was conducted on 9 mathematics teachers. 7 problems were designed. Semi structured interview and content analysis was used. The results indicated that properties of quadrilaterals which are best known by participating teachers were square and rectangles. The another results of the study was that teachers correctly defined properties related angles and sides quadrilaterals but have problems with properties related diagonals. In addition to this results some teachers were not classify quadrilaterals, and the teachers who use hierarchical classification were not fully established family relations.*

Key Words: Perceptions of Quadrilaterals, Classification of Quadrilaterals

THEORETıCAL FRAMEWORK

Concepts are quite important in mathematics. In order to express the entire cognitive structure about concept, Tall and Vinner (1981) have used the term "concept image" and stated that concept image includes mental image about the concept, properties and processes about the concept. When considered in the sense of geometry, there are 3 different situations about geometrical concepts. These are definition, image and properties of the shape of geometrical concept. Definitions are structured in a way that would features minimum information of the properties of concept. It was aimed for definitions as the basic principle that they would be economically short and understandable (De Villiers, 1998; Fujita, 2012). Apart from the definitions of concepts, every geometrical concept has a visual image. Visual image might be one step further than the concept. In this sense, typical (prototype) sample are the key factor. Properties of concepts are key factors in the context of making relations between concepts, differentiating concepts, making generalizations of concepts and finally making classification.

Classification of quadrilaterals is regarded important in making relations between quadrilaterals and therefore in solving problems and proof studies in geometry. Because if a quadrilateral is in the same family with another quadrilateral than solutions, proofs, properties etc. raised for this quadrilateral would be valid for the other one as well. In this sense, De Villiers (1994) points out that individuals can make two types of classification of quadrilaterals. The first one is hierarchical classification which done by relating quadrilaterals under subset according to the properties they have. The other one is partition classification which means classification of quadrilaterals in separate sets individually according to the properties they have. According to de Villiers (1994), hierarchical classification was defined as

a type of classification which makes family relations of quadrilaterals more understandable. As various researchers have explained, making hierarchical classification of individuals and dominating education in this sense should be supported (De Villiers, 1994; De Villiers 1998; Fujita, 2012).

When the literature is studied, there are various studies about perception of quadrilateral nearly in every age group. In these studies it is seen that students are addressed questions about quadrilaterals, listing their properties, making separation and relation between them and making classification.

In the studies which are carried out for understanding of quadrilaterals by naming the drawn quadrilaterals or drawing the named quadrilaterals; different types of square, rectangle, rhombus and trapezium were asked to students in different age group who participated in the study. As a result of these studies, it was determined that quadrilaterals which have prototypical drawing in general were to a large extent marked or named correctly (Okazaki and Fujita, 2007; Fujita and Jones, 2007; Fujita, 2012; Clements, Swaminathan, Hanibal and Sarama, 1999; Monaghan, 2000).

The role of definition is very important for perception of quadrilateral. In the literature studies, personal definitions which were made free from academic definition were considered in understanding the perception so, the studies were focused on how the quadrilaterals were defined individually. In the study of Fujita and Jones (2007), it was found that the percentage of correct answer of prospective class teachers about quadrilaterals is quite low.

In the studies in which were asked to classify quadrilaterals, it was detected that great majority of the students make partition classification (Erez and Yerushalmy, 2006; Monaghan, 2000; Fujita and Jones, 2006; Berkün, 2011). The reason was stated as the images of quadrilaterals and the effect of these images on inclusion relations of quadrilaterals.

In the study of Okazaki and Fujita (2007) (with students aged between 15 and 18 and prospective class teachers) in which it was analyzed how the students make relations among quadrilaterals, it was determined that around 50% students gave correct answers for the question of whether rhombus and rectangle are parallelogram or not. Moreover, they have answered the view that square is rectangle and rhombus with quite a low percentage such as 35%. There are different studies which states that the relation between rectangles are defined correctly in low rates (Okazaki and Fujita, 2007; Elia, Gagatsis, Deliyianni at al., 2009; Heinze and Ossietzky, 2002; Fujita and Jones, 2007; De Villiers, 1998). In these studies it was found that not being able to make relation between square-rhombus and parallelogram-rectangle result from the fact that their angles are not 90^0. Similarly, the relation between square-rhombus and parallelogram-rectangle could not be done since their edge lengths are not same and they were included in different family.

Perception and classification of geometrical shapes contribute solution of problems both in real life and in different fields of mathematics (NCTM, 2004; Martin and

Strutchens, 2000). As it's understood from the mentioned studies, it was observed that individuals belonging to different age groups don't have difficulty with the recognition of quadrilaterals, yet it was observed that they have difficulty with the classification of quadrilaterals and with the understanding of relations between them. However, the maths teachers and primary school teachers have great responsibility in the correct description of quadrilaterals by the individuals, and relating them to each other and classification of them by the individuals. Maths teachers have an important role in especially perceiving the relations of quadrilaterals and classification of them, which necessitates higher cognitive abilities. That's why, the knowledge of maths teachers about quadrilaterals must be checked in detail. Moreover, when the literature is investigated, there is no study showing the knowledge of maths teachers about quadrilaterals. For this reason, it is expected that this study will compensate for the loss in the literature.

THE PURPOSE OF THE RESEARCH

In this study it was aimed at determine how mathematics teachers comprehend the properties of quadrilaterals, how they construct the relationship between quadrilaterals and classify them.

METHOD

In this study, semi-structured interview was done with the method of qualitative research. In the interviews participants are asked seven questions five of them are open-ended and two of them are multiple choice questions. The interview questions were prepared based on the related literature. Questions were composed of three sections. In the first section teachers were asked how they define basic features of quadrilaterals, in the second section how they relate quadrilaterals in pairs, in the third section questions were about the family relations between quadrilaterals. Sample questions about each section are in the Appendix 1. Interviews were done with 9 middle school mathematics teachers who work at 8 different schools at a city in Turkey. Three of these teacher's teaching experience is less than five years, of the two between five and ten years, of the other two between twenty and ten and of the last two have more than twenty years of experience. According to the year of teaching experience the ages of the teachers range from twenty-five to fifty. The two teachers who participated in the research graduated from training institute, which had a two-year education and the others graduated from an education faculty, which has a four-year education years. All the interviews were recorded with audio recorder. Before the data was analyzed, each audio record was fully transcribed into verbal data. Content analysis technique was used in the analysis of obtained data.

FINDINGS

In this study, data were obtained by middle school mathematics teachers, analyzed under three main titles such as perceptions of quadrilaterals and understanding of the relations of quadrilaterals and classification of quadrilaterals.

Mathematics Teachers' Perceptions of Quadrilaterals

In the study, teachers' knowledge about the properties of square, rectangle, parallelogram, rhombus and trapezium was questioned.

It was determined that teachers correctly define properties of parallelism and diagonal of square and rectangle as well as properties of angle and vertices. Only one teacher was not sure in defining diagonal property of rectangle.

In data about parallelogram, the first perception of teachers was the parallelism of vertices. As it is understood from the statement of a teacher, *"if it is parallelogram, then their opposite vertices are parallel"* that since the name includes the term parallel, the first perception was parallelism.

It was observed that teachers' knowledge about properties of angle and vertices of parallelogram are mostly correct but they have doubts about properties of diagonals. This irresolution is related with the equivalence of lengths and angle bisector.

The first perception of teacher about rhombus is the equivalence of vertices due to the name of quadrilateral. The view of a teacher suitable about this condition was:

"We teach it as a quadrilateral whose all vertices are equal. Even when defining this quadrilateral, we reflect as it diamond shape colloquially." (T7)

As it is understood from the statement of T7, teachers identify rhombus with diamond shape. Similarly, it was observed that other teachers use same statements.

Moreover, similar to parallelogram, it was observed that teachers' knowledge about properties of angle and vertices of rhombus are mostly correct but they have doubts about properties of diagonals. This irresolution is related with the equivalence of lengths and angle bisector.

It was determined that teachers have two different views about the shape of trapezium. It was found that this diversity of views results from "parallelism of sides" and it was defined as the parallelism of "at least two sides" or "only 2 sides". Views of two different teachers:

"Only two sides of trapezium can be parallel" (T6)

"According to the property of trapezium (hesitates), at least two opposite sides should be parallel." (T8)

It was observed that teachers have problem with angle properties. However most of the teachers (7 teachers) stated that "side angles which are not parallel are supplementary angles" others (2 teachers) stated that "consecutive angles are supplementary angles".

It was determined that some of the teachers both know the properties of trapezium and have different perception about formal status of trapezium. Statements about this view are:

"Trapezium is the form of rectangle whose sides have different cut" (T2)

"Considering it as the divided form of triangle but right and left sides are not parallel like the sides below and above." (T7)

When the explanations of Ö2 and Ö7 are analyzed, it is observed that they relate trapezium with the images of other quadrilaterals in the sense of form.

Mathematics Teachers' Understanding of the Relations of Quadrilaterals

Teachers who participated in the study were asked to make bilateral relations among quadrilaterals. Teachers making relations of between square-rectangle, rectangle-parallelogram, square-parallelogram, square-rhombus were determined. According to data, it was observed that most of the teachers focus on common properties and differences rather than classification.

For the common and different properties of square and rectangle, it was observed that they focus on angle, vertices and diagonal properties. Only one teacher make the relation that square is a special kind of rectangle. The statement of this teacher is as such:

"For example students are asked. Is square a rectangle or not? Most of the students say it is not. But I emphasize that square is a rectangle, specially." (T6)

The statement of a teacher among 8 others who regard the relation between square and rectangle in the sense of common and different properties is as such:

"... there are short sides and long sides in rectangle as well. But all the sides of square are equal, yet opposite sides of rectangle are equal. Diagonals of rectangle do not intersect perpendicularly but diagonals of square do so. All the sides of rectangle are not equal but all the sides of square are."(T3)

As it is understood from the statement of T3, they make relation between square and rectangle by focusing on the differences of sides and diagonals. Moreover, from this statement it can be concluded that teacher first of all visualizes rectangle as a quadrilateral which has short side and long side.

Properties of angle and diagonal were mentioned for the common and different properties of rectangle and parallelogram. However, properties of diagonal were stated wrong by most of the teachers. For example, a teacher pointed out a wrong property by saying **"***diagonal length of both shapes is equal." (T4)*

Apart from the answers which are based on common and different properties of rectangle and parallelogram, there are teachers who make family relation as well. One of these teachers gave this answer.

"... while teaching rectangle, we mention that it is a special kind of parallelogram. We make such a relation while teaching rectangle in order to show that we can obtain it out of a parallelogram." (T8)

In addition to these relations, a teacher made a relation based on the image rather than the properties of rectangle and parallelogram. The answer of this teacher was:

"... think that we make a rectangle pattern from modeling clay, when we pull from opposite corners equally we obtain a parallelogram, so we can make a parallelogram out of a rectangle" (T3)

A teacher who participated in the study stated there would be no relation between rectangle and parallelogram with this sentence.

"All the properties of rectangle are different from parallelogram, they do not match with each other." (T5)

Angle and side properties were mentioned for the common and different properties of square and parallelogram. There is one teacher who makes family relation for square and parallelogram. This teacher gave such an answer:

"... it is valid for parallelograms. Because it is parallelogram. Rectangle, square are parallelograms. We should give this message." (T6)

3 teachers who participated in the study made no relation between square and parallelogram. One of these teacher stated that:

"All the properties of square cannot be valid for parallelograms, we cannot make a relation because there are too many incompatible properties." (T3)

Angle and diagonal properties were mentioned for the common and different properties of square and rhombus.

For the relation based on angle properties, it was stated that if all the angles are right angle that the rectangle is square, if not then it is rhombus. There is one teacher who makes family relation (T6). In addition to these relations, two teachers made a relation based on the image rather than the properties of square and rhombus. Answers of these teachers are:

"the rectangle when you press one side of the square is rhombus." (T1)

"If we pull the sides of square to one side equally then it is rhombus. We should pull the sides equally because opposite sides are equal." (T3)

Mathematics Teachers' Classification Of Quadrilaterals

Teachers were asked to classify quadrilaterals in the study. According to data obtained from teachers' answers, there are 3 situations for quadrilaterals classification. These are:

1. Not making any relation among quadrilaterals (T1, T2, T5)

2. Partition classification of quadrilaterals: Making a relation with a table only according to common and different properties. (T4, T7)

3. Hierarchical classification of quadrilaterals: Making relation with specific family relations. There are 3 different schemes for this situation.

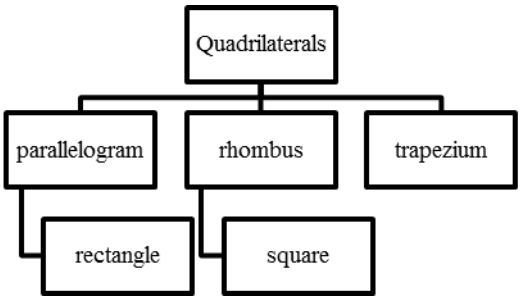

Figure 1: (T3, T8)

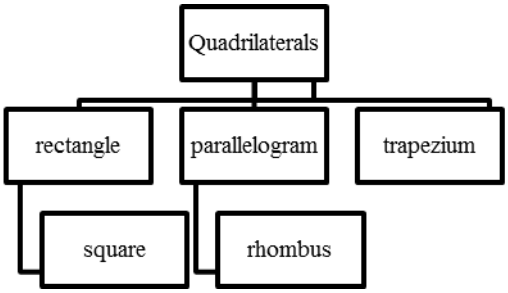

Figure 2: (T9)

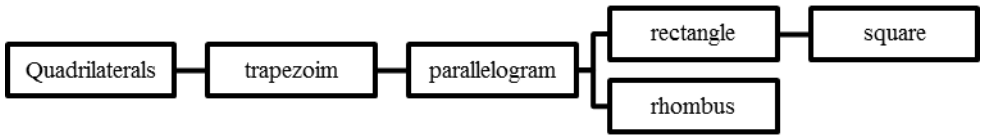

Figure 3: (T6)

In Figure 1, it is seen that teachers interpret square as a special kind of rhombus, but not a special kind of rectangle; interpret rectangle as a special kind of parallelogram but cannot interpret rhombus as a special kind of parallelogram. This results from the fact that teachers make interpretation based on side lengths. In this way, they could make relation between square and rhombus whose side lengths are equal and between rectangle and parallelogram whose only opposite side lengths are equal.

In Figure 2, it is seen that teacher interpret square as a special kind of rectangle and also interpret rhombus as a special kind of parallelogram but cannot interpret square as a special kind of rhombus. This condition results from the fact that teacher makes interpretation based on angles. In this way, they make relations between square and rectangle whose angles are 90º and between parallelogram and rhombus whose angles are not 90º.

When the figures for the third situation were analyzed, it is seen that teachers cannot make family relations totally. It was seen that teachers who have made Figure 1 and Figure 2 separates trapezium from quadrilaterals but teacher who made Figure 3 includes trapezium into the classification with other quadrilaterals. Since teachers generally do not consider property of parallelism, this causes them to ignore the fact that square and rectangle are parallelogram. It was stated that only 3 teachers interpret square or rectangle as parallelogram.

CONCLUSION

Generalization was not done in this study which was carried out with 9 mathematics teacher; yet in the light of data, specific profiles for the teachers' perception of quadrilaterals were put forward. It was determined that teachers who participated in the study correctly define properties related angles and sides of quadrilaterals but have problems with properties related diagonals. It was observed that although teachers have problems about properties related diagonals of quadrilaterals, they

focus rather on diagonal properties while establish relationship between quadrilaterals. However, it was seen that they define properties of diagonal wrongly. In Monaghan's (2000) study, in which he presented children's views (aged 11-12 years) of the differences between some quadrilaterals; he concluded that the images, properties related sides and the angles are focused on for differences between quadrilaterals, but not properties related diagonals.

The geometrical figures among mentioned quadrilaterals which is least known by participating teachers was "trapezium". It was observed that teachers have difficulty in identifying the image of trapezium. This condition was observed in the study of Berkün (2011) which was carried out with earlier ages.

While parallelism is not prominent for square, rectangle and rhombus in the findings, it was stated among the properties of parallelogram and trapezium. Similar findings were observed in the studies of Fujita (2012), Fujita and Jones (2007), Okazaki and Fujita(2007), Heinze and Ossietzky (2002).

Three of the teachers who participated in the study could not make classification. Contrary to most of the studies (Berkün, 2011; Monaghan, 2000; De Villiers, 1994), the some of the teachers (4 teachers) could make hierarchical classification. However, they could not correctly and fully establish family relations. The reason of this was that teachers focused on only angle and side properties, could not interpret and relate all the properties together.

REFERENCES

Berkün, M. (2011). *İlköğretim 5 ve 7. Sınıf Öğrencilerinin Çokgenler Üzerindeki İmgeleri ve Sınıflandırma Stratejileri*. Yayınlanmamış Yüksek Lisans Tezi, D.E.Ü.

Clements, D. H., Swaminathan, S., Hanibal, M. ve Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education*, *30* (2), 192- 212.

De Villiers, M. (1994). The Role and Function of a Hierarchical Classification of Quadrilaterals. *For the learning of mathematics*, 14, 11-18.

De Villiers, M. (1998). To Teach Definitions in Geometry or Teach to Define?. In A.Oliver ve K. Newstead (Eds.), *Proceedings of The 22nd Conference of the International Group for the Psychology of Mathematics Education* 2, 248-255.

Elia, I., Gagatsis, A., Deliyianni, E., Monoyiou, A. ve Michael, S. (2009). A Structual Model of Primary School Students' Operative Apprehension of Geometrical Figures. M. Kaldrimidou ve H. Sakonidis (Eds). *Proceedings of The 33rd Conference of the Internatıonal Group for the Psychology of Mathematics Education*, 3, 1-8.

Erez, M. M. ve Yerushalmy, M. (2006). ''If You Can Turn A Rectangle Into A Square, You Can Turn A Square Into A Rectangle ...'' Young Students

Experıence. *International Journal of Computers for Mathematical Learning 11*, 271–299.

Fujita, T. (2012). Learners' Level of Understanding of Inclusion Relations of Quadrilaterals and Prototype Phenomenon. *The Journal of Mathematical Behavior, 31*: 60-72.

Fujita, T. ve Jones, K. (2006). Primary Trainee Teachers' Understanding of Basic Geometrical Figures in Scotland. J. Novotana, H. Moraova, K. Magdelena ve N. Stehlikova (Eds). *Proceedings of The 30th Conference of the International Group for the Psychology of Mathematics Education,* V.3, 14-21.

Fujita, T. ve Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: towards a theoretical framing, *Research in Mathematics Education*, 9(1&2), 3-20.

Heinze, A. ve Ossietzky, C. (2002). "…Because a Square is not a Rectangle" Students' Knowledge of Simple Geometrical Concepts When Starting to Learn Proof. In A. Cockburn ve E. Nardi (Eds.): *Proceedings of The 26th Conference of the International Group for the Psychology of Mathematics Education*, V.3, 81-88.

Martin, W. G., ve Strutchens, M. E. (2000). Geometry and Measurement. In E. A.Silver & P. A. Kenney (Eds.), Results from the Seventh Mathematics Assessment of the National Assessment of Educational Progress (193–234). Reston, VA: National Council of Teachers of Mathematics.

Monaghan, F. (2000). What Difference Does It Make? Children's Views of the Differences Between Some Quadrilaterals. *Educational Studies in Mathematics*, *42*(2),179-196.

Okazaki, M., ve Fujita,T. (2007) . Prototype Phenomena and Common Cognitive Paths in the Understanding of the Inclusion Relations Between Quadrilaterals in Japan and Scotland. H.Woo, K.Park ve D. Seo (Eds): *Proceedings of The 31st Conference of the Internatıonal Group for the Psychology of Mathematics Education*, 4, 41-48.

Tall, D., ve Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity. *Educational Studies in Mathematics,* 12(2), 151-16.

APPENDIX 1.

1) Question Aiming at Description of Quadrilaterals:

A teacher asked his students the features of a trapezium. The answers of five students are as follows:

Student 1: The two opposing sides are parallel.

Student 2: The consecutive angles are complimentary.

Student 3: There are two types of trapezium. Right trapezium and isosceles trapezium.

Student 4: The high of a right trapezium is the one which is perpendicular to the paralel sides.

Student 5: The domain(alan) of each trapezium is the multiplication its height with the half of the total lenght of lower base and upper base.

When you look into the answers, which ones are correct and which ones are wrong? Why?

2) Question Aiming at Relations of Quadrilaterals:

Answer the questions below:

a) What is difference between a sguare and a rectangle?

b) What is difference between a rectangle and a parallelogram?

c) What is difference between a sguare and a rhombus?

d) What is difference between a trapezium and a parallelogram?

3) Question Aiming At Classification:

Below are five items about quadrilaterals.Which one is correct?

a) All the features of a rectangle are the same for all squares.

b) All the features of squares are the same for all rectangles.

c) All the features of a rectangle is the same for all parallelogram.

d) All the features of squares are the same for all rhombus.

e) None of the choices above is correct.