USING ORIGAMI TO ENHANCE GEOMETRIC REASONING AND ACHIEVEMENT

Sevil ARICI Fatma ASLAN-TUTAK

Boğaziçi University, Turkey

This paper was part of a broader study that examined the effect of the instruction on spatial visualization, geometric reasoning and geometry achievement. This paper focused on the geometric reasoning and geometric achievement. Participants were 167 high school students in Turkey. In a pre-test/post-test design, the experimental group was instructed with the origami-based instruction and the control group was instructed with the traditional instruction for four weeks. Geometric Reasoning Test and Geometry Achievement Test were prepared to measure geometric reasoning and geometry achievement, respectively. The results of the repeated measures Analysis of Variance on test scores showed that the origami-based instruction had a statistically significant effect on geometric reasoning and geometry achievement.

Key-words: origami, geometric reasoning, geometry achievement, high school geometry

INTRODUCTION

Reasoning, proving, creativity and problem solving are involved in Turkish high school geometry curriculum. These skills are expected to be developed through effective geometry instruction. For an effective geometry instruction, a closer look into the geometry learning may be necessary. Three theoretical frameworks may give insights in understanding geometry learning of students. Duval (1998) argues that geometric thinking combines three cognitive processes which are *visualization*, *construction*, and *reasoning*. Visualization is keystone for geometry instruction since students should be able to identify geometric figures in different dimensions in order to reach conclusions about geometric entities. Duval's (1998) cognitive processes formed the mainstay of the test design of our study. Besides, Smith (2010) asserts that geometric thinking is based on *proving*, *justifying*, and *argumentation*. Smith's (2010) categorization of geometric thinking was used in preparing the scoring rubric of the Geometric Reasoning Test (GRT). Furthermore, as a developmental model on geometric thinking, the van Hiele (1959/1985) theory states that geometric thinking progresses in hierarchical stages. Van Hiele stages were considered in the design of the instruction of the existing study. These three theoretical frameworks all agree on the use of manipulative materials in teaching geometry for effective learning of abstract concepts and relationships. The use of manipulatives in teaching geometry and mathematics is also suggested in the literature (e.g. Dorier, Gutiérrez, & Strässer, 2003; Sriraman & English, 2005). Manipulatives can be useful in facilitating students' progression to higher levels of geometric thinking. Thus, origami, the art of paper folding can be used in teaching geometry considering its manipulative nature.

Origami was remarked as a beneficial tool in geometry education in a wide age range of learners by many authors (e.g. Boakes, 2009; Çakmak, 2009; Golan, 2011; Hull, 2006; Pope & Lam, 2011; Sağsöz, 2008; Winckler, Wolf, & Bock, 2011). Origami is connected with many mathematical and geometric concepts and principles such as angle bisectors, fractions, division, ratio, triangles, polygons, congruence, and symmetry. Origami is also referred as a trigger for proof and strong mathematical arguments (Pope & Lam, 2011; Winckler et al., 2011) so that origami may be used to develop students' geometric reasoning and geometry knowledge.

Although origami is recommended by many authors as a useful instructional tool, research on the use of origami in schools is limited (Boakes, 2009; Çakmak, 2009; Sağsöz, 2008). Besides, the research involving origami is concentrated on primary and middle school students (e.g. Boakes, 2009; Çakmak, 2009). Therefore, there exists a need for investigating the effect of origami on high school geometry education. Thus, the focus of this paper was to investigate the effect of origami-based instruction on tenth-grade students' geometric reasoning and geometry achievement in a Turkish high school.

METHOD

Settings

The Turkish education system has kindergarten, primary, middle, secondary and higher education levels. The secondary school level consists of 4-year education (9^{th} to 12^{th} grade). There are seven types of high schools: Science High School, Anatolian Teacher High School, Anatolian High School, Social Sciences High School, Fine Arts and Sports High School, General High School, and Vocational High School. After middle school, there is an entrance exam to high schools except general and vocational high schools. Students who can enter to Science High Schools have the highest scores in the entrance exam. However, students in the General High Schools and Vocational High Schools have the lowest scores in the high school entrance exam. Thus, the academic achievement of Vocational High School and General High School students was generally low compared to other types of high schools.

A common curriculum, which is determined by the Turkish Ministry of Education (MEB), is implemented in all high school types. Students select their courses according to their interests and orientations. Science-Mathematics Orientation (SM), Turkish-Mathematics Orientation (TM), Turkish-Social Orientation (TS), and Foreign Language Orientation (FL) are the types of orientations that students select at the 10^{th} grade. Students with SM orientation have courses like Mathematics, Geometry, Physics, Chemistry and Biology. Students with TM orientation also have Mathematics and Geometry lessons but do not have Science courses. Students with TS orientation do not have Science courses, either.

The study was conducted among 167 10^{th} graders of a General High School in Turkey. The students were chosen from all three different academic orientations: SM, TM, and TS. There are 10 classrooms of 10^{th} graders in the school. One TS classroom was excluded because the instructor was different than the instructor of other nine classrooms. Two classes of each orientation were randomly selected. Then, one of the two chosen classrooms with each orientation was randomly assigned to be in the experimental group and the remaining class was taken to be in the control group. So, the experimental group and the control group each have three classes with three different academic orientations.

Treatment

Students were instructed on a geometry unit (triangles) for four weeks based on the curriculum objectives of the Ministry of National Education (MEB). The control group received traditional instruction, which followed the 10th grade MEB textbook. On the other side, the experimental group received origami-based instruction, which contained origami activities in addition to following the same textbook with the control group. Besides, the same teacher (the first author) instructed both the experimental and control groups. Each week, the control group had two lesson hours for the geometry instruction whereas the experimental group had three lesson hours since one extra lesson hour was used for instructions on folding. The topics for the instruction were the basics of triangles, angle and side relationship in triangle, angle bisectors, medians, perpendicular bisectors, and altitudes in triangles.

Week	Topic	Origami activity
1	Basics of triangles, classifying triangles	Dart; equilateral triangle
2	Angle and side relationship in a triangle	Swan
3	Angle bisectors in a triangle	Whale
4	Altitude, median, and angle bisector relationships, perpendicular bisectors	Folding a square paper

Table 1: Origami-based instruction

Table 1 shows the origami activities used in the origami-based instruction. The origami activities were developed considering the related literature (e.g. Hull, 2006). Lesson plans were prepared for both groups (Arıcı, 2012). In the origami-based instruction, students initially folded the origami models and then they were guided by the instructor to make the necessary geometric relationships and proofs. In the traditional instruction, students were also guided to make proofs about triangles but no origami activities were used in guiding students in the process of proving.

Figure 1 presents an example of origami activities in the origami-based instruction. Folding an equilateral triangle is based on Hull (2006). To fold an equilateral triangle, students were initially told to fold and unfold a square paper in half. Then, the teacher wanted students to fold a bottom corner (the D point) up to the fold line and mark the point on the line (the point C). After this, students were guided to fold the equilateral triangle (ACD). Folding the equilateral triangle lasted approximately 15 minutes. After folding, students were questioned about geometric relationships. They were asked why the folded triangle was an equilateral triangle (ACD). As a hint, students were asked to find the angle measures and side lengths of the triangles that were formed after folding. Students had to recognize the congruence of the triangles (ADG and ACG) in order to prove the triangle ACD to be an equilateral triangle (see Figure 2).

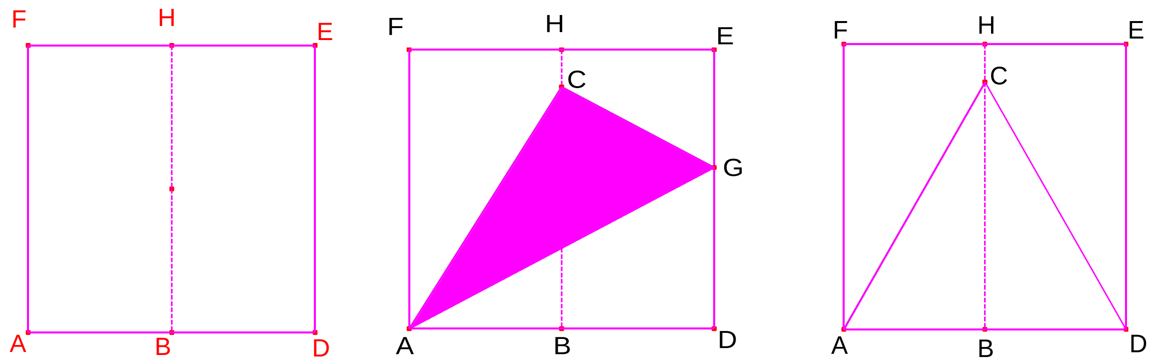


Figure 1: Folding an equilateral triangle

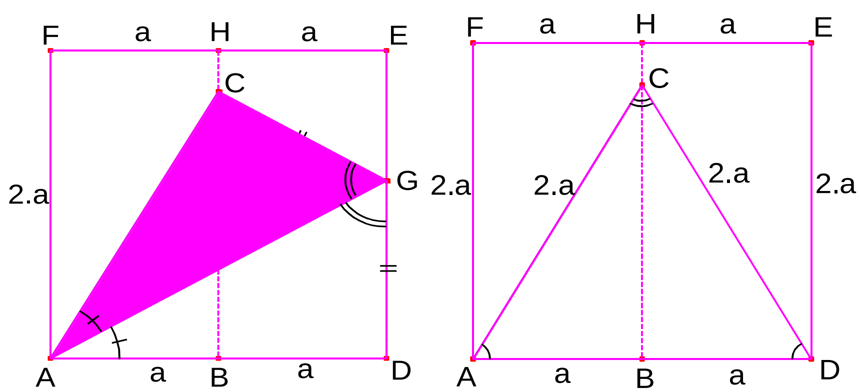
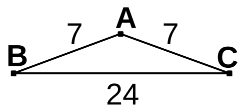


Figure 2: Steps to prove that the folded triangle was an equilateral triangle

Instruments

The pre-tests were administered to all students a week before the geometry instruction. After the instruction, post-tests were administered. The Geometric Reasoning Test (GRT) was prepared to assess students' geometric reasoning abilities related with triangles. Besides, the Geometry Achievement Test (GAT) was formed to assess students' geometry achievement concerning triangles that were instructed during the study. GRT and GAT were piloted before the administration of the tests. After the instruction, the GAT was administered and the GRT was administered one week later from the GAT.

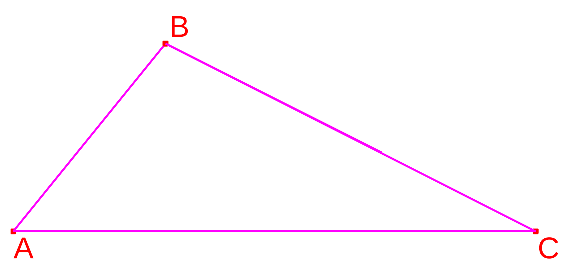
The GAT was developed based on the objectives of the curriculum concerning triangles. There were 11 open-ended items in the GAT. Parallel forms of the GAT were prepared as pre-test and post-test (Figure 3, a sample item from the post-test). The GAT was administered in 45 minutes (one lesson hour). A scoring rubric for the GAT was generated considering the solution steps of the questions. Items in the GAT contain at most four steps so that the maximum score for an item is given as 4. The Cronbach alpha for the GAT was 0.86.



Is it possible that the triangle ABC in the figure with length sides of 7, 7, and 24 units exist? Explain.

Figure 3: An item of the Geometry Achievement Post-test (translated from Turkish)

The GRT was prepared based on the literature about proof and construction questions including triangles (e.g. Jacobs, 2003). Duval's (1998) framework was used as a basis to form the items in the GRT. Duval's (1998) cognitive process of construction includes using tools and GRT has items based on construction. Items were about isosceles triangle, congruence of triangles, angle bisectors, inscribed and circumscribed circles of triangles, angle and side relationships. There were 13 open-ended items in the GRT (Figure 4, a sample item). The maximum score for an item in the GRT is 2 points because an item has at most two steps of the solution. Moreover, the Cronbach alpha for the GRT was 0.70. Scoring of the items in the GRT was based on the Smith's (2010) framework of reasoning. Responses which were aligned with the *argumentation* were given the lowest score in the GRT. Besides, responses which were aligned with the *proving* were given the highest score in the GRT. Proving contains a comprehensive explanation of geometrical expression. Justifying gives some reasons but these reasons are not sufficient to explain the geometric expression. Argumentation also involves explanation but this explanation is not necessarily accurate or geometrical.



How can you draw the incircle of a triangle ABC using lines? Draw the incircle and explain your answer.

Figure 4: An item of the GRT (translated from Turkish)

RESULTS

A repeated measures of Analysis of Variance (ANOVA) was done on each test. Data were analyzed to investigate whether there was any statistically significant difference between mean pre-test scores, between mean post-test scores, between mean pre-test and post-test scores of the experimental group and the control group. The significance level was kept on 0.05 during the analyses. The between-subjects factor was group (control or experimental) and the within-subjects factor was time (pre-test or post-test).

Descriptive statistics for the Geometry Achievement Test (GAT) and Geometric Reasoning Test (GRT) are shown in the Table 2. At the beginning, there were 90 students in the experimental group and 94 students in the experimental group. However, there were missing students during the test time so the missing ones were excluded from the analyses. GRT was administered after GAT so that the number of students who took the tests differed.

Test	Group	$\overline{X}$ (pre-test)	SD	$\overline{X}$ (post-test)	SD
GAT	Control (N=88)	1.52	2.54	5.95	6.79
	Experimental (N=79)	2.13	3.56	12.80	11.05
GRT	Control (N=90)	3.79	1.97	4.79	2.16
	Experimental (N=77)	3.14	2.38	6.42	3.78

Table 2: Means and standard deviations for the tests

According to the repeated measures ANOVA on the Geometry Achievement Test (GAT) scores, the interaction effect between time and group ($F(1,165)=25.02$, $p<0.001$, $\eta_p^2=0.13$), the main effect of group on GAT ($F(1,165)=19.53$, $p<0.001$, $\eta_p^2=0.11$), and the main effect of time on GAT were statistically significant ($F(1,165)=146.60$, $p<0.001$, $\eta_p^2=0.47$). Moreover, the pairwise comparisons for GAT by group and time indicated that there was not a statistically significant mean difference between pre-test scores for the experimental group and the control group ($F(1,165)=1.61$, $p=0.206$, $\eta_p^2=1.61$) but there was a statistically significant mean difference between post-test scores for the experimental group and the control group ($F(1,165)=23.75$, $p<0.001$, $\eta_p^2=23.75$). The results also revealed that the mean difference between post-test and pre-test was statistically significant for the experimental group ($F(1,165)=0.01$, $p<0.001$, $\eta_p^2=0.46$) and for the control group ($F(1,165)=26.68$, $p<0.001$, $\eta_p^2=0.14$). Besides, the mean difference between post-test and pre-test was higher in the experimental group than that in the control group.

According to the repeated measures ANOVA on the Geometric Reasoning Test (GRT) scores, the main effect of time on GRT ($F(1,165)=102.86$, $p<0.001$, $\eta_p^2=0.38$) and the interaction effect between time and group ($F(1,165)=29.10$, $p<0.001$, $\eta_p^2=0.15$) were statistically significant. However, the main effect of group on GRT was not statistically significant ($F(1,165)=1.97$, $p=0.162$, $\eta_p^2=0.01$). Furthermore, the pairwise comparisons for GRT by group and time indicated that there was not a statistically significant mean difference between pre-test scores for the experimental group and the control group ($F(1,165)=3.68$, $p=0.057$, $\eta_p^2=0.02$) but there was a statistically significant mean difference between post-test scores for the experimental group and the control group ($F(1,165)=12.06$, $p<0.005$, $\eta_p^2=0.07$). The results also showed that the mean difference between post-test and pre-test was statistically significant for the experimental group ($F(1,165)=0.01$, $p<0.001$, $\eta_p^2=0.40$) and for the control group ($F(1,165)=12.22$, $p<0.005$, $\eta_p^2=0.07$). Moreover, the mean difference between post-test and pre-test was higher in the experimental group than that in the control group.

CONCLUSION AND DISCUSSION

The results suggested that the origami-based instruction could have an effect on students' geometry achievement and geometric reasoning concerning triangles.

The results about geometry achievement revealed that there was a statistically significant change in geometry achievement scores of students, who received origami-based instruction, from pre-test to post-test time. The geometry achievement scores of students who received traditional instruction also showed a statistically significant change from pre-test time to post-test time. However, the average difference in geometry achievement scores from pre-test time to post-test time for students who received the origami-based instruction was more than that for those who received the traditional instruction.

Although there were some studies that found no significant effect of origami on students' geometry knowledge (e.g. Boakes, 2009), most teachers and authors recommended using origami in geometry teaching to enhance students' geometry knowledge (Golan, 2011; Pope & Lam, 2011). The difference of the test types might have affected the results concerning geometry achievement. For example, Boakes (2009) used a multiple-choice test to measure geometry achievement. But, our study used a geometry achievement test with open-ended items that were prepared in parallel with the curriculum objectives. Besides, the grade level might have affected the results concerning geometry achievement. Boakes' (2009) study involved middle-school students as the sample but our study used high-school students. The design of the instruction also had an effect on students' geometry achievement. The origami-based instruction combined Duval's (1998) cognitive processes of visualization, construction, and reasoning. Such an instruction might have facilitated students' knowledge about the related topic. Golan (2011) reported that using origami in geometry lessons aligned with van Hiele theory helped students to develop their geometry knowledge for higher levels of abstraction. The existing study also took into consideration the van Hiele theory in designing the geometry lessons using origami activities. Besides, Pope and Lam (2011) noted that origami was a good way to enrich school curriculum by providing opportunities for problem solving and creativity. Aligned with the literature, the results suggested that origami could be an additional source of instruction to enhance geometry knowledge of high school students.

The results about geometric reasoning were similar to those about geometry achievement. The results concerning geometric reasoning presented a statistically significant change in geometric reasoning scores of participants, who received origami-based instruction, from pre-test time to post-test time. The geometric reasoning scores of students who received traditional instruction also showed a statistically significant change from pre-test time to post-test time. However, the average difference in geometric reasoning scores from pre-test time to post-test time for students who received the origami-based instruction was more than that for those who received the traditional instruction.

Geometric reasoning involves proving (generating logically correct deductive arguments as Smith (2010) suggested) so that it requires a higher level of geometric thinking. Origami as a tool might have facilitated students to reason at a higher level of abstraction in geometry. The connection of origami with geometric reasoning was also stressed in many resources (e.g. Pope & Lam, 2011; Winckler et al., 2011). For instance, Pope and Lam (2011) presented proof examples that used origami to show that origami could be an important context to develop reasoning. Furthermore, Winckler and his colleagues (2011) stated that origami was an enjoyable way to teach geometric principles to high school students as a bridge between theory and practice. In parallel with previous publications related to origami and reasoning, the results of our study pointed out that origami-based instruction could promote high school students' geometric reasoning related with triangles.

There are certain limitations in our study. For example, classrooms of students were randomly chosen as to be in the experimental or the control group. However, students themselves were not randomly chosen because the school policy did not allow changing students' classrooms.

The effect of origami-based instruction on participants' geometry achievement and geometric reasoning mentioned in this paper implied that origami could be incorporated in geometry lessons. Teachers and curriculum planners should also take into consideration the benefits of integrating origami in high school geometry instruction. Origami is not just for fun but also may be a meaningful context for high-level thinking in geometry.

REFERENCES

Arıcı, S. (2012). *The effect of origami-based instruction on spatial visualization, geometry achievement and geometric reasoning of tenth-grade students* (Master's thesis). Boğaziçi University, Istanbul, Turkey.

Boakes, N.J. (2009). Origami instruction in the middle school mathematics classroom: Its impact on spatial visualization and geometry knowledge of students. *Research in Middle Level Education*, *32*(7), 1-12.

Çakmak, S. (2009). *An investigation of the effect of origami-based instruction on elementary students' spatial ability in mathematics* (Master's thesis). Middle East Technical University, Ankara, Turkey.

Dorier, J.-L., Gutiérrez, A., & Strässer, R. (2003). Introduction. In *Thematic working group 7 of the Third Conference of the European Society for Research in Mathematics Education (CERME 3): Geometrical thinking* (pp. 1-10). Bellaria, Italy.

Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Eds.), *Perspectives on the teaching of geometry for the 21st century: An ICMI study* (pp. 37-52). Dordrecht: Kluwer.

Jacobs, H.R. (2003). *Geometry: Seeing, doing, understanding* (3rd ed.). New York, NY: Macmillan.

Golan, M. (2011). Origametria and the van Hiele theory of teaching geometry. In P. Wang Iverson, R. J. Lang & M. Yim (Eds.), *Origami 5: Fifth international meeting of origami science, mathematics and education (5OSME)* (pp. 141-150). Boca Raton: CRC Press.

Hull, T. (2006). *Project origami: Activities for exploring mathematics*. Natick, MA: A K Peters.

Pope, S., & Lam, T. K. (2011). Origami and learning mathematics. In P. Wang Iverson, R. J. Lang & M. Yim (Eds.), *Origami 5: Fifth international meeting of origami science, mathematics and education (5OSME)* (pp. 205-218). Boca Raton: CRC Press.

Sağsöz, D.A. (2008). *İlköğretim 6. sınıflardaki kesirler konusunun origami yardımıyla öğretimi* (Master's thesis). Atatürk University, Erzurum, Turkey.

Smith, R.C. (2010). *A comparison of middle school students' mathematical arguments in technological and non-technological environments* (Doctoral dissertation). North Carolina State University, North Carolina, United States.

Sriraman, B., & English, L. D. (2005). On the teaching and learning of Dienes' principles. *International Reviews in Mathematics Education (ZDM)*, *37*(3), 258-262.

Van Hiele, P.M. (1959/1985). The child's thought and geometry. In D. Fuys, D. Geddes, & R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele* (pp. 243-252). Brooklyn, NY: Brooklyn College, School of Education.

Winckler, M. J., Wolf, K. D., & Bock, H. G. (2011). Hands-on geometry with origami. In P. Wang Iverson, R. J. Lang & M. Yim (Eds.), *Origami 5: Fifth international meeting of origami science, mathematics and education (5OSME)* (pp. 219-232). Boca Raton: CRC Press.