PSYCHOLOGY STUDENTS' STRATEGIES AND SEMIOTIC CONFLICTS WHEN ASSESSING INDEPENDENCE

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In this paper we analyse the strategies used in assessing independence in a two-way contingency table in a sample of 414 psychology students in three different Spanish universities. Strategies are analyzed from the point of their complexity level, their correctness, and the semiotic conflicts involved in the student's reasoning. Although there was perfect independence in the data, most students provided a moderate-sized association coefficient and a positive judgment of association. Few strategies reached the highest complexity level and a number of semiotic conflicts were identified.

INTRODUCTION

Although contingency tables are common to present statistical information and association judgment is a priority learning issue in statistics courses (Zieffler, 2006), little attention is paid to its teaching, in assuming that its interpretation is easy.

This paper describes part of a wider research that was aimed to assess the students' understanding of association in contingency tables before teaching and compare this knowledge with that acquired by the students after a teaching sequence designed for this research (Cañadas, 2012). In the initial assessment three different items corresponding to direct association, inverse association and independence were used. In this paper we only analyze the students' strategies in the item corresponding to perfect independence, because this was the item where students were less accurate in estimating the association coefficient and moreover, the majority of students considered there was association in the data (Batanero, Cañadas, Estepa & Arteaga, 2011). Strategies are classified according to the levels defined by Pérez-Echeverría (1990), and according to its mathematical correctness. Results are compared with those described by Batanero, Estepa, Godino and Green (1996) and students' semiotic conflicts that may explain their wrong strategies are described.

PREVIOUS RESEARCH

Research on association was started by Inhelder and Piaget (1955), who described the strategies used at different ages when judging association in tasks that were formally equivalent to a 2x2 contingency table (see Table 1).

Later psychological studies were developed with adults, and showed that subjects perform poorly, when judging association in these tables. For example, Smedlund (1963) found that some adults base their judgment only on cell *a* in Table 1 or by comparing *a* with *b*. Allan and Jenkins (1983) showed the tendency to base the association judgments on the difference between cell *a* and *d* in Table 1. Although Allan and Jenkins (1983) found that comparing the diagonals *(a+d)* and *(c+b)* was common in adults. Jenkins and Ward (1965) remarked that this strategy of is only valid in tables that have rows with equal marginal frequencies. A correct strategy valid for all types of tables, according to Jenkins and Ward is comparing of the two conditional probabilities, *P(B|A)* and *P(B|Not A)*, that is, comparing *a/(a+c)* with *b/(b+d).* Pérez Echevarría (1990) classified strategies that have been identified in psychological research until that time into 6 levels of performance. Levels 0 to 3 correspond to students who use 0 to 3 cells to perform the association judgment. In levels 4 and 5 the students use the four cells; the difference is that comparison between the cells are additive in Level 4 and multiplicative in Level 5.

	A	Not A	Total
B	a	b	a+b
Not B	c	d	c+d
Total	a+c	b+d	

Table 1: Format of a 2x2 contingency table

In a different perspective, Chapman (1967, pp. 151) reported a common bias that he called "illusory correlation": "*the report by observers of a correlation between two classes of events which, in reality, (a) are not correlated, (b) are correlated to a lesser extent than reported, or (c) are correlated in the opposite direction from that which is reported*". Many researchers have reported this bias and suggest that previous theories disturb the estimates of association (e.g., Wright & Murphy, 1984; Meiser & Hewstone, 2006). Batanero, Estepa, Godino, & Green (1996) analyzed the performance of 213 17 year-olds high school students and their strategies in association judgments and defined different conceptions of association: (a) *causal conception* according to which the subject only considers association between variables, when it can be explained by the presence of a cause - effect relationship; *unidirectional conception*, by which the student does not accept an inverse association, and *local conception*, where the association is deduced from only a part of the data.

Our research is aimed to go further in the analysis of strategies in the case where data show perfect independence, since this was the task in which students performed worse in Batanero et al. (1996) study, and in our previous study (Batanero, Cañadas & Arteaga, 2011). We also complement the type of analyses made in previous research in order to identify potential *semiotic conflicts* of students when assessing independence. This term is taken from, who adapted from Godino, Batanero, and Font (2007), who adapted from Eco (1979) the idea of semiotic function: "the correspondences (relations of dependence or function) between an antecedent (expression, signifier) and a consequent (content, signified or meaning), established by a subject (person or institution) according to certain criteria or a corresponding code" (p. 130). These authors also suggest that in mathematical practices different objects intervene: *problems, actions, concepts-definition, language properties and arguments*. For them "*the role of representation is not exclusively undertaken by language":* in accordance with Peirce's semiotic, they assume that "*the different types of objects can also be expression or content of the semiotic functions"* (p. 103). The authors term *semiotic conflict,* any "*disparities between the student's interpretation and the meaning in the mathematics institution"* (p. 133). This construct is weaker than that of conception, as stability is not required from the student, but only a misunderstanding or misinterpretation of a mathematical concept, property, language or procedure.

METHOD

The sample included 414 students in their first year of psychology studies from three Spanish universities: Almeria (115 students), Granada (237 students) and Huelva (62 students), all of them taking an introductory statistics course. The task given to the students (Figure 1) was adapted from Batanero et al. (1996), changing to a context of psychological diagnose, without varying the data. The samples included all the students enrolled in the course and attending the session; the difference is sample sizes was due to the size of the University. Though they had not yet studied association in the course they were following, the students had studied statistics in secondary education.

A researcher is studying the relationship between stress and insomnia. In a sample of 250 people he observed the following results:

	Stress disorders	No stress disorders
Insomnia	90	60
No insomnia	60	40

Looking to these data, do you think there is a relationship between stress and insomnia?

Please mark on the scale below a point between 0 (minimum strength) and 1 (maximum strength), according the strength of relationship you perceive in these data.

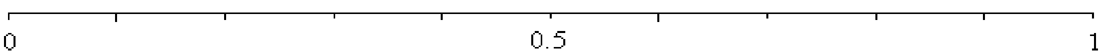

Figure 1: Task proposed to the students

In part (a) of each item, students are asked to provide an association judgment. There are two categories of responses: (a) the student considers that the variables in the item are related (judging association); (b) the student considers the variables to be not related (judging independence). The estimation for the association coefficient estimation is deduced by measuring the exact position of the point drawn by the student on the numerical scale (0-1) in the second part of the item. Finally, a qualitative analysis was used to identify the strategies used by the students and their semiotic conflicts. The classification of strategies was performed by two different members of the team; in case of disagreement, it was discussed with the other team members until an agreement was reached.

RESULTS AND DISCUSSION

Association judgment and estimation of association

In Table 2 we present the percentage of students who considered (or not) the existence of association between the variables and the mean value of their estimation for the association coefficient. Most students indicated the existence of association although data in the item correspond to perfect independence. This judgment is consistent with their estimation of the association coefficient, which is moderate-sized in average. The differences in mean estimate (in the ANOVA test) or in the percent of students judging association (in the chi-square test) between the three universities were no statistically significant. This suggests that students' responses were similar, despite the difference in educational context. Results may be explained by illusory correlation (Chapman, 1967) since in this item data contradicts the students' previous theories (that stress is related to insomnia), as well as by the causal conception of association, reported by Batanero et al. (1996), who found 55.4% of students judging association in an item with the same numerical data and where the data also contradicted previous theories.

	Almería (n=115)	Granada (n=237)	Huelva (n=62)	Total (n=414)
Mean estimate	0.51	0.47	0.44	0.47
Number (and percent) of students considering there is association in the data	87 (75.7)	194 (81.9)	42 (67.7)	323 (78.1)

Table 2: Mean estimates of association and association judgment

Level of strategies

In order to explain the above results we analyzed the students' strategies, and the mathematical practices involved in these strategies. Firstly, the students' strategies were classified according to their correctness, in three groups: (a) correct strategies (that always produce a correct association judgement); for example, comparing the proportion of people with and without stress disorder in both groups of people with and without insomnia; (b) partly correct strategies that produces a correct response only for specific tables and involves some correct (although partial) ideas about association; for example comparing the sums of diagonals in the table; is only valid when the marginal frequencies in the table are identical; (c) incorrect strategies, when students use a procedure that is a priori wrong in all type of tables; such as for example, using only cell *a* to solve the task. This classification was crossed with the levels of difficulty proposed by Pérez Echeverría (1990), in the following way:

Level 0 Strategy. The student used no information from the table and only took into account his/her own previous theories about the association; the illusory correlation (Chapman & Chapman, 1969; Murphy & Medin, 1985) is shown in this case, for example: *Since when you do not sleep this cause some stress* (Strategy 0, Student 5).

Level 1 Strategy. When the student only used one cell in the table; usually cell *a* because this is the cell when both characters are present and has a higher impact on our attention (Smedlund, 1963; Beyth & Maron, 1982; Shaklee & Mins, 1982, Yates & Curley, 1986): *"there is association, since 40% of the sample have insomnia and stress"* (Strategy 11; Student 111); other students used one of the cells *b* or *c* that contradicts the association: *"there is no relation since there are 60 people with stress and no insomnia and this is a big percentage"* (Strategy 12, Student 51).

Level 2 Strategy. Some students used two cells; for example, they compared *a* with *b* or *a* with *c*, so that they deduced association from only one conditional distribution, which is incorrect: *"If you look to the people with insomnia, there are more people with stress (90) than without stress (60)"* (Strategy 21, Student 21). Other students compared the cells with maximum and minimum frequency (Batanero et al., 1996): *"there are 90 people with stress and insomnia and 40 without stress and without insomnia; 90>40, but the relation is not too strong"* (Strategy 22, Student 61).

Level 3 Strategy: In this strategy students used three cells; for example, they compared cell *a* with *b* and *c*. In general, these students discarded cell *d* that correspond to the absence of both characters (Batanero et al., 1996): *"there is relationship as there are more people with stress and insomnia (90 people) and exactly the same number (60) with either stress and no insomnia or insomnia and no stress"* (Strategy 3, Student 153). All level 1 to 3 strategies are incorrect as the students only use part of the data and then show a local conception of association (Batanero et al., 1996), while part of the strategies in levels 4 and 5 are partly correct or correct. In Level 4 and 5 strategies students use all the cells; for example when they compare rows or columns. The difference is performing additive or multiplicative comparisons.

Level 4 strategies are based on additive comparisons of the four cells. One example is comparing the sum of diagonals *(a+d)* with *(b+c): "there are 130 people with both stress and insomnia or no stress and no insomnia, while there is only 120 with one of these symptoms.* (Strategy 41, Student 176). This strategy was found by Allan and Jenkins (1983). This strategy could provide a good solution when the marginal frequencies (number of people with and without insomnia) were equal, according to Shaklee (1983); for this reason we considered the strategy to be partly correct. In another example, students compared two conditional distributions in additive way: *In people with insomnia there is a difference of 30 having stress, while the difference in people without insomnia is smaller (20)"* (Strategy 42, Student 267) or else compared all the absolute frequencies among them: "*There are many with stress and insomnia (90) but the relationships is not strong, since having stress and no insomnia or insomnia and no stress (69) is also high, much higher than no insomnia and no stress (40)* (Strategy 43, Student 156).

Level 5 strategies use all the four cells with multiplicative comparisons, but still may be incorrect or partly correct. For example, a wrong strategy is to compute all the joint relative frequencies and compared them: *"I computed the percent of each data and compare the results:* $\frac{90}{250}100 = 36\%$ *;* $\frac{60}{250}100 = 24\%$ *;* $\frac{60}{250}100 = 24\%$ *;* $\frac{40}{250}100 = 16\%$ (Strategy 51, Student 11). This procedure is incorrect, because the association should be deduced from conditional distributions and not from joint distributions. An example of partly correct strategy is assuming that all joint relative frequencies in the table should be identical (that is, 25% cases in each cell). We considered this strategy partly correct because the student computed some "expected" frequencies, compared them with the observed frequencies and deduced that there was no association because these two types of frequencies were different. The strategy could have worked (and then was partly correct) in case the computation of expected frequencies were correct: "*I divided 250 between 4 (25%) to see the number of people we should expect in each cell, in case of no relationship. However, although the number of people with no insomnia and stress and no stress and insomnia are close to 25% there is a big difference in the other cells; more than 25% people with both insomnia and stress and less than 25% people with none of them* (Strategy 52, Student 1).

Finally, among the level 5 correct strategies we find students who compared two conditional distributions; for example, *a/(a+b)* with *c/(c+d)* or else performed a similar procedure in comparing columns: "*When we observe the table, 60% of people with insomnia have stress and also 60% or people with no insomnia have stress; the percentage is the same"* (Strategy 53, Student 28). Another correct strategy is comparing possibilities in favour and against B for each value of A; which was described by Batanero et al. (1996): *There are 90 people with insomnia for every 60 with no insomnia when you have stress; that is the odds are 3/2; the same odds 60/40 apply when you do not have stress"* (Strategy 54, Student 21).

	Incorrect	Partly correct	Correct	Total
Level 0	13 (100.0)			13 (3.1)
Level 1	73 (100.0)			73 (17.6)
Level 2	108 (100.0)			108 (26.1)
Level 3	27 (100.0)			27 (6.5)
Level 4	27 (26.2)	76 (73.8)		103 (24.9)
Level 5	20 (26.3)	10 (13.2)	46 (60.5)	76 (18.4)
No response	14 (100.0)			14 (3.4)
Total	282 (68.1)	86 (20.8)	46 (11.1)	414 (100)

Table 3: Frequency (and percent of total) of strategies by level

In Table 3 we present the frequency of responses in the above categorization. Only 11.1% of students used correct strategies and 20.8 % of them partly correct strategies. Students tended to use either level 2 or level 4 strategies none of which are correct, and moreover there were a big percentage of students who did not use all the cells information, since their strategies were level 3 or lower. At level 4, about 30% students compared join frequencies among them, an incorrect strategy described by Batanero et al. (1996) and about 60% used the four cells with additive comparisons, a strategy described by Inhelder and Piaget (1955) in the concrete-operation level but that also was found by Batanero et al. (1996) in high school students. Finally most of level 5 strategies were correct as students either compared the odds ratios or compared conditional distributions a strategy proposed by Jenkins and Ward (1965) and also found in Batanero et al. (1996). The accuracy in the estimation of the association coefficient (true value is equal to zero) increases with the strategy correctness (Table 4) and the differences were statistically significant in the Anova test.

p value	Incorrect strategy		Partly correct strategy		Correct strategy	
(Anova)	Mean	Typical error	Mean	Typical error	Mean	Typical error
0.000	0.536	0.012	0.432	0.024	0.174	0.038

Table 4: Mean estimate of association coefficient in different type of strategy

Semiotic conflicts

The analysis of incorrect and partly correct strategies led to the identification of the several semiotic conflicts, which are classified below, according to whether they involve disparities in the meaning students assigned to association or independence (attributing wrong properties to these concepts) or other conflicts:

1. Incorrect properties assigned to association:

(a) Identification of causality and association, which was called *causal conception of association* by Batanero et al, (1996) was found in level 0 strategies. Since we could not observe the stability of this belief, we just considered it as a wrong interpretation of a property of association (a semiotic conflict), because although causality always involves association, association does not always involves causality, students misinterpreted that this relation was symmetrical.

(b) Assuming that association can be deduced from only a part of the data (which was called *local conception of association* by Batanero et al, 1996 and also appear in previous research, e.g. Smedlund, 1963; Beyth & Maron, 1982; Shaklee & Mins, 1982; Yates & Curley, 1986; Pérez Echeverría, 1990). This conflict appears in all the strategies in levels 3 or below; again, we interpret this belief as a misinterpretation (conflict) as we could not observe its stability.

(c) Assuming that it is possible to deduce association from additive comparisons, a strategy that should have been overcome, according to Inhelder and Piaget (1951) at the formal operations stage, but which, however, appeared in our sample in all the level 4 strategies. Students using this procedure only took into account the favourable cases (and not all the possible cases) when comparing probabilities; and therefore this strategy involves a conflict in understanding the idea of probability.

(d) Assuming that association can be deduced from only one conditional distribution (Strategy 21), which was also described by Inhelder and Piaget (1951). Students here only used the conditional distribution of *B* given *A* and did not identify the relevance of the conditional distribution of *B*, given not *A* for the association.

(e) Assuming that the difference in absolute conditional frequencies is enough to support association, an error which was found in Smedlund (1963) and Shaklee and Mins (1982) and appear in Strategy 43. The conflict appears as students misunderstood the important of relative frequencies in the study of association.

(f) Assuming that the association can be deduced from the difference between the sums of diagonals in the table. That strategy was considered to be correct by Piaget and Inhelder, but Allan & Jenkins (1983) and Shaklee and Tucker (1980) suggested it does not work in the general case (for example, in the task given to the students). It appeared in strategy 53.

(g) Assuming that $a>d$ in case of association, which appeared in Strategy 22. These students considered that only these two cells influence the association and consequently they did not understood that cell *d*, have the same value than *a* on the association. We did not find this strategy in previous research.

2. Incorrect properties attributed to independence

(a) Expecting equal join frequencies in case of independence, which involves confusion between the ideas of independence and equiprobability. It appeared in strategy 52 and was not described in previous research.

3. Other conflicts: Basing the association judgment in their own opinion, instead of considering the data, which appear in level 0 strategies, where students were guided by the illusory correlation phenomenon described by Chapman & Chapman (1969).

DISCUSSION AND TEACHING IMPLICATIONS

Most psychology students in our study judged association in a task where there was perfect independence, due to the illusory correlation phenomenon and their previous theories, which affected their accuracy in estimating the association coefficient. Results in our study were worse, as compared with Batanero et al. (1996) since a higher percent judged association. These authors did not inform about the estimate of association by their students; in our study the estimation was consistent with the association judgment. Results were very close in all participating universities. Regarding the conceptions described by these authors, we observed the causal and the local conception. Since we could not check the stability of these conceptions, we used instead the idea of semiotic conflict, which only involves a mistaken interpretation of a mathematical expression, a concept, property or procedure. Our list of semiotic conflicts is wider than the list of conceptions described by the authors, as new conflicts related to the ideas of association and independence were identified in our study. For example, in addition to assume that association may be judged from only part of the data, another frequent conflict was assuming that association can be judged from absolute frequencies (instead of relative frequencies). Another new conflict is assuming that the cells in the four cells in the table should have equal frequency in case of independence. Since semiotic conflicts do not assume a strong conviction on the part of the students, it is possible to change them with adequate instruction and then the identification of these conflicts in the students is a first step in order to correct their wrong reasoning and improving their competence in judging association.

All these reasons and our results suggest the need for further research about teaching association, since the causal conception and the effect of illusory correlation does not seem to improve with traditional instruction (Batanero, Godino, & Estepa, 1998). Our purpose is to continue this work by designing an alternative teaching with activities that confront students with their conflicts and help them overcome them. This proposal will be tested and students will be assessed in order to compare their intuitive ideas with those acquired as a result of teaching.

Acknowledgements

Research supported by the project: EDU2010-14947, grant FPU-AP2009-2807 (MCINN- FEDER) and Group FQMN-126 (Junta de Andalucía).

References

Allan, L. G., & Jenkins, H.M. (1983). The effect of representations of binary variables on judgment of influence, *Learning and Motivation, 14,* 381-405.

Batanero, C., Cañadas, G., Estepa, A. & Arteaga, P. (2011) Psychology students' estimation of association. *Statistique et Enseignement*, *3* (1), 3-18.

Batanero, C., Estepa, A., Godino, J. D., & Green, D. (1996) Intuitive strategies and preconceptions about association in contingency tables. *Journal for Research in Mathematics Education, 27*(2), 151-169.

Batanero, C., Godino, J. D., & Estepa, A. (1998). Building the meaning of statistical association trough data analysis activities. Research Forum. In A. Olivier & K, Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (v.1, pp. 221-242). University of Stellembosch.

Beyth-Marrow, R. (1982). Perception of correlation reexamined. Memory and cognition, *10* (6), 511-519.

Chapman, L. J. (1967). Illusory correlation in observational report. *Journal of Verbal Learning and Verbal Behavior, 6*(1), 151-155

Cañadas, G. (2012). *Comprensión intuitive y aprendizaje formal de las tablas de contingencia en alumnus de psicología* (Intuitive understanding and formal learning of contingency tables by psychology students). Ph.D. University of Granada,

Chapman, L. J. & Chapman, J. P. (1969). Illusory correlation as an obstacle to the use of valid psychodiagnostic signs, *Journal of Abnormal Psychology*, 74, 271-280.

Eco (1979). *Tratado de semiótica general* (General semiotics). Barcelona: Lumen.

Erlick, D.E., & Mills, R.G. (1967). Perceptual quantification of conditional dependency, *Journal of Experimental Psychology, 73,* 1, 9-14.

Font, J. D., Godino, J. D., & D'Amore, B. (2007). An ontosemiotic approach to representations in mathematics education. *For the Learning of Mathematics*, *27* (2), 3-9**.**

Godino, J. D. Batanero, C., & Font, V. (2007) The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education,* 39 (1-2), 127-135.

Inhelder, B., & Piaget, J. (1955). *De la logique de l´enfant à la logique de l´adolescent.* (From the child's logic to the adolescent's logic). Paris: Presses Universitaires de France.

Jenkins, H. M., & Ward, W. C. (1965). Judgment of the contingency between responses and outcomes, *Psychological Monographs, 79,* 1-17.

Meiser, T., & Hewstone, M. (2006). Illusory and spurious correlations: Distinct phenomena or joint outcomes of exemplar-based category learning? *European Journal of Social Psychology*, *363*(3), 315-336.

Pérez Echeverría, M. P. (1990). *Psicología del razonamiento probabilístico.* Madrid: Ediciones de la Universidad Autónoma de Madrid.

Schield, M. (2006). Statistical literay survey analysis: reading graphs and tables of rates and percentages. En B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics.* Cape Town: International Statistical Institute and International Association for Statistical Education. On line: http://www. stat.auckland.ac.nz/~iase.

Shaklee, H. (1983). Human covariation judgment: accuracy and strategy. *Learning and Motivation, 14,* 433-448.

Shaklee, H., & Mins, M. (1982). Sources of error in judging event covariations: Effects of memory demands, *Journal of Experimental Psychology Learning, Memory and Cognition, 8*(3), 208-224.

Smedlund, J. (1963). The concept of correlation in adults. *Scadinavian Journal of Psychology,* 4, 165-174.

Wright, J.C, & Murphy, G. L. (1984). The utility of theories in intuitive statistics: the robustness of theory-based judgments, *Journal of Experimental Psychology General, 113*(2), 301-322.

Yates, J. F. & Curley, S. P. (1986). Contingency judgment: Primacy effects and attention decrement, *Acta Psicológica, 62*, 293-302.

Zieffler, A. (2006). *A longitudinal investigation of the development of college students' reasoning about bivariate data during an introductory statistics course.* Unpublished PhD. University of Minnesota.