CONTEXTUALIZING SAMPLING – TEACHING CHALLENGES AND POSSIBILITIES

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The aim of the present paper is to shed light on mathematical knowledge for teaching probability. In particular we investigate critical instances when a teacher tries to keep track on the idea of sampling and random variation by allocating the discussion to an everyday context. The analysis is based on a certain episode of a longer teaching experiment. The analytical construct of contextualization was used as a means to provide structure to the qualitative analysis performed. Our analysis provides insight into the nature and role of teachers' knowledge of content and teaching. In particular, the study suggests the idea of a meta-contextual knowledge that teachers need to develop in order to keep track of the intended object of learning when allocating their teaching to an everyday context.

Keywords: Sampling, probability, descriptive statistics, mathematical knowledge for teaching, knowledge of content and teaching

INTRODUCTION AND AIM OF STUDY

This paper is part of a larger research project, which aims at investigating the nature and role of *Mathematical Knowledge for Teaching Probability* (MKTP).

Mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008) is a general knowledge framework which aims at structuring an understanding of teacher competences for teaching mathematics. The framework is divided in two main domains, *subject matter knowledge* (SMK) and *pedagogical content knowledge* (PCK*)*. Both domains are viewed equally important for teaching mathematics. On an overall level, the notion of SMK implies that all mathematical content matter is similar in nature, following a certain mathematical logic and structure of axioms. However, there are reasons to consider specifics of MKT in relation to probability theory as this is considered different in nature to other branches of mathematics.

Steinbring (1991) argues that not even the basic concepts of probability theory fit into a logical deductive way of acquiring mathematical knowledge. For example, the concept of probability is used to define randomness but what is unique with probability theory is that the inverse is also true. In this way, probability concepts are self-referential, which implies that teaching probability requires a dynamic outlook on knowledge development. Stohl (2005) also recognizes that probability-laden situations could not be structured in a purely logical and deterministic way as for instance algebra and geometry. Since probability deals with modeling random dependent situations it is not possible, by certainty, to derive and predict a certain result.

Connected to the specific nature of probability, there are reasons to take a closer look at the pedagogical content knowledge teachers should develop in trying to make sense of probability in teaching. However, what specific set of demands that is put on mathematics teachers to handle probability in the classrooms is an uncharted territory. Against this background, the present paper aims to contribute with insights on *knowledge of content and teaching*, a certain sub-category of PCK in the MKT framework, in relation to the teaching of probability. Knowledge of content and teaching is described by Ball et al. (2008) as a repertoire of forms of representations, techniques and examples and the interaction between mathematical understanding and pedagogical assets in relation to students' learning. In particular the paper focuses on the challenges a teacher might meet when contextualizing *sampling* in an everyday context through classroom discussion.

Described by Gal (2005), there are two overall reasons for teaching probability in school. The first is internal in that probability is part of mathematics and should be developed in its own right. The second reason is external in the sense that "the learning of probability is essential to help to prepare students for life" (Gal, 2005, p. 39). Elaborating on this duality, Gal (2005) suggests that teaching should put sufficient emphasis on probability-laden situations in the real world in order to help students to develop *probability literacy*. However, even if Gal (2005) proposes that the teaching of probability should, to great extent, stress real world applications of probability discussions, little is known about how such teaching should be structured and the nature of MKTP required in such teaching, for making details of intended probability content available to the students learning.

The teacher's role is seen crucial for orchestrating a classroom discussion in ways that allow students to make sense of the mathematics (McCrone, 2005). To make a story of a mathematic content available to the students and to support their meaning makings of that story, teachers should make details of the content explicit in the discussions, in both explanations and questions asked (Franke, Kazemi, & Battey, 2007). In the present paper we intend to look at a teacher's explications or lack of explications, when the teacher tries to make the story of sampling available for a class of Grade 5 students in a Swedish school by allocating the discussion to an everyday context.

Aim of Study

The aim of the present paper is to shed light on knowledge of content and teaching probability (KCTP) required for *contextualizing* a probabilistic content in an everyday story. In particular we intend to investigate critical instances when a teacher tries to make sense of the idea of sampling by allocating the discussion to an everyday context.

FOCAL PROJECT, CONTEXT AND CONTEXTUALIZATION

The analysis reported in the present paper was based of the analytical construct of contextualization (Halldén, 1999; Nilsson, 2009). The construct of contextualization can be considered a constructivist reaction to constructivists' purported neglect of contextual aspects of learning.

Context plays a certain role in contextualization. In a sociocultural perspective, context refers to stable physical and discursive elements of a setting in which a learning activity takes place (Resnick, 1989). However, based on constructivist assumptions, in the present paper context refers to the cognitive context shaped by the learner's personal interpretations of an activity (Cobb, 1986). To speak about students' processes of contextualization is to speak about how learners struggle to render a phenomenon or concept intelligible and plausible in personal contexts of interpretation (Caravita & Halldén, 1994). This idea rests on the principle that we always experience something in a certain way, from a certain set of premises and assumptions (Säljö, Riesbeck, & Wyndhamn, 2003). Related to that, contextualization gives sense to learning mechanisms of how and why different knowledge elements make the activation of others either more or less likely (cf. Shelton, 2003). In other words, talking about how teachers and students contextualize a phenomenon is a way of organizing and conceptualizing their views of the phenomenon and what these views imply for their understanding of the phenomenon and their way of communicating the phenomenon.

To clarify how different contextual elements serve as points of reference in processes of contextualisation, it has shown fruitful to structure between elements referring to a *conceptual, situational* and *cultural* context. The conceptual context refers to personal constructions of concepts and subject matter-structures. The situational context refers to interpretations made in the interaction between the individual and the immediate surroundings, including interpretations of figurative material, possible actions and directly observable sensations. Third is the *cultural* context, referring to constructions of discursive rules, conventions, patterns of behaviour and other social aspects of the environment (Halldén, 1999).

It has also shown to be fruitful to structure between the *focal project* (FP) of a reasoning process and the *context* in which the FP is treated or contextualized (Nilsson, 2009; Nilsson & Ryve, 2010). To talk about a FP means that we pay serious attention to what a teacher or a student is actually trying to make sense of, or trying to work out. So, what constitutes the FP in a pattern of contextualization is the problem, goal or intention that a teacher or a student engage in and interpret as being their obligation to solve or achieve (Nilsson, 2009). Hence, viewing meaning making as a process of contextualization an analyst strives to account for agents' FPs and the way they contextualize these projects in order to understand how and why content elements are brought into play in a reasoning.

METHOD

Shaughnessy (2003) emphasizes the role of data experimentation in the teaching and learning of probability. He claims that, when trying to make sense of data, students are encouraged to develop probability questions. However, from research we know that it is not an easy task for students to make sense of data. Makar and Rubin (2009) report that students often disregard frequency data and also are struggling to connect conclusions to the data they have collected regardless of sample size (Makar & Rubin, 2009). The authors point to the need for teachers to make this connection more explicit to the learners. Also, Pratt, Johnston-Wilder, Ainley, and Mason (2008) highlight students' difficulties interpreting information in a data sample as they challenged students (10-11 years old) to infer, from data, the unknown configuration of a virtual die. Similar to Stohl and Tarr (2002), Pratt et al. (2008), show that what became critical for the students was the role of sample size, to understand the difference between drawing conclusions on the basis of the global (long-term) and local (short-term) behaviour of a data sample.

The rationale of a teaching experiment

The episode that will be presented and analysed in the present paper concerns how a teacher introduces a discussion on sampling and sample size through an everyday contextualization. The episode emerged within the frame of a teaching experiment in which a class of fifth graders had first been planting pumpkins and sunflowers. Since this is the first time these students encounter probability theory in the school environment, we find it important to give a brief overview of the teaching that took place before the episode that is in focus of our analysis in order to better understand our interpretations of the teachers' and the students' acts.

A developmental team, consisting of Maria, the teacher of the class, the second author of the paper and Torsten, a specialist on outdoor education, stood behind the design of the experiment. Maria did all the teaching. Ten students participated in the study, which was stretched over two lessons. Each student was assigned an individual box of one square metre during the first lesson in which they were planting 18 pumpkin and 18 sunflower seeds. Maria held a probability lesson about three week after the seeds were planted, which was based on the germination of pumpkins and sunflower seeds. The teaching rationale of the activity was, at the first place, to challenge the students on the idea of sampling and, specifically, on the role of sample size for making probability predictions. The teacher tackled this issue by confronting the students with the variations in growing outcomes between the students' individual boxes. For instance, in one box only one of the 18 pumpkins grew. This was compared to boxes were up to eleven of the 18 pumpkins grew. Based on this, the teacher led a discussion about how we have to combine more and more observations in order to get a more valid result of the chance for a seed to grow. The first part of the teaching ended with putting together all student results and dividing the resulting number by the total number of planted seeds to get an estimation of the probability for a seed to grow. The same procedure was done for both the pumpkins and the sunflowers.

The rationale of the second part of the teaching episode was to represent the results of growing seeds in the structure of a two-way contingency table (Figure 1).

	Growing	Not growing	Total
Sunflowers	51	129	180
Pumpkins	67	113	180
Total	118	242	360

Figure 1: Two-way table of the growing of sunflowers and pumpkins

Not only the teaching but also the work of the developmental team were audio and video recorded. The developmental team made brief reflections on the teaching in immediate connection to the teaching. However, the detailed analysis presented in this paper was made by the authors of the present paper.

This paper focuses on a certain episode of the entire teaching were Maria ends the lesson by telling an improvised story containing a sampling situation. The story is supposed to wrap up the discussions and introduce the coming lesson, in which the students were supposed to plan for and conduct their own probability-laden, statistical investigation. The third lesson was not actually supposed to be part of the teaching experiment. But implicitly it was, as the introduction of the lesson was included at the end of the second lesson.

RESULTS AND ANALYSIS

We enter the teaching when Maria portrays a picture where she waits outside a store for her husband to exit. While waiting, she keeps track of the number of each sex that visited the store.

The same context but competing intentions

Maria portrays a situational context in which she is standing outside a store one early morning, reflecting on how many males that have visited the store. The intended aim is to contextualize sampling in a new way so the students get the opportunity to translate what has been discussed previously about samples. Within this situational context Maria introduces a *genus context,* which is directed towards the distribution of men and women taking responsibility of shopping for groceries. Taking a close look at the way in which Maria is orchestrating the story, we note that she includes multiple FPs to her story, which we consider making it hard for the students to follow the storyline and the mathematical idea of it. We particularly distinguish two specific conflicting FP, dealt with within the same genus oriented context. First Maria has the intention of contextualizing sampling to the students and second she has the intention to convince her husband to more often do the shopping. Implicitly the story is also meant to create an interest for similar surveys since Maria is planning for an upcoming assignment on the topic of statistics and probability. She struggles to explicate the mathematical intention and give support to the students' meaning making process early in the story.

Maria: Another male, that makes three men, and it continued. I wondered, are men such early birds? My husband usually doesn't go shopping this early. I'm going to tell him that I stood here and watched. Then when I've counted ten people, seven of them were male. So I thought I would tell him that seven out of ten were male.

[…]

Maria: The worst part was, he [the husband] didn't exit. So I kept on waiting, and then a lot of women started to appear

Student 1: And then, when you looked out the window, you saw a girls' bus and a boys' bus.

Maria: No, but there were women and women and women. Eventually 20 men and 20 women had exit the store.

Our interpretation is that Marias mathematical intention has not been made plain for the students so far. Instead they are subjected to the intention to reduce the disparity between men and women in typical household choirs like shopping for groceries. Since this is a mathematics lesson, there are reasons to believe that the students are searching for a mathematical intention in the story of which they should make sense. We interpret that one student has caught on such a meaning and that there exists an unofficial game between Maria and her husband. If there are more men shopping in the morning, Marias husband must do so more often as well and she wins. So the student tries to challenge the rules and prerequisites with a statement.

Maria: And do you know what I did then? I stopped counting.

Student 1: Coward!

Maria: You know, I didn't want there to be more girls than boys, right?

Student 2: You didn't have to tell him [Marias' husband] that you counted.

Maria: No, I actually didn't, since the outcome was what it was.

[…]

Maria: If I had stood there and counted a hundred, maybe 70% would have been women, right?

[…]

Maria: And if I would of stood there even longer I would of gotten an even better result. What the probability was for a man or a woman entering the store, right?

The teacher does not show any signs of recognizing the fact that there are competing intentions in the discussion. We note that more students seem to try to make sense of the story in terms of a game. They sense that Maria is about to loose, calling her a coward and offering her a way out, and she is admitting that fact from their point of view by not telling her husband about the result. To think of the story in terms of a game rather than whether men shop for groceries or not, might fit better with the students' expectations of probability application and their attempt of meaning making. In other words, since Marias competing intentions appear confusing, the students contextualize the intended learning object of the story in a way that makes more sense to them.

Maria keeps on pushing her genus perspective combined with fairness. She does not want it to be more girls than boys shopping for groceries in an equal society. There is also a preconception on the teacher's behalf that the students share this view. But randomness does not have a genus agenda, making the two intentions conflicting, so she steps in and manipulate the result by keeping the sample size small. Her intuition tells her that more women shop for groceries, since that mental picture is more available to her. The session illuminates that Maria has a sound understanding of sampling and the role of sample size. Because of her knowledge of sampling she acknowledge that it becomes a risk to continue the thought experiment. She anticipates that the result will probably not differ much from the assumed true value in the long run, opposing her gender equality agenda.

There are more references to Marias mathematical intention towards the end of the story. She eventually manages to explicate the mathematical content in a statement formulated with probability wording relevant to sampling by posing the question "What the probability was for a man or a woman entering the store, right?". The situational and cultural context seems to be the focus of Maria, instead of the conceptual context, for most of the episode. Next we will analyse what motivates the story.

Competing driving questions

Marias original driving question for her improvised investigation outside the store could be interpreted in different ways. It could be a question of descriptive statistics; how many men shop for groceries early in the morning? Or a more probability oriented question; what is the probability for a man going shopping for groceries early in the morning? Since the driving question is left unspoken, it is left up to the students to formulate a question that fits their individual meaning making of the story. We suggest that the close relation between statistics and probability when handling relative frequencies becomes an obstacle for teachers. It places great demand on teachers' subject content knowledge to be explicit about what is the FP to be investigated, e.g., if the FP is to offer a description of a phenomenon or if it is to make a prediction of future behaviour of the phenomenon. More concretely, is the FP to describe differences between how many women and men that are shopping for groceries or is the FP to estimate the probability that the next person out of the store is a man? We interpret that Maria her self struggles between the two as she continues the story.

Maria: If I had been there a whole day, I would of known if there are more women or men that goes shopping that specific day. And if I had been there for two days, I would of gotten an even better result. And if I would be staying there even longer I would have had an even better result: what the probability was for a man or a woman entering the store, right?

One way of interpreting Marias words is to question her subject matter knowledge, that she is not sure her self about the difference between descriptive statistics and probability theory or even if there are any difference. But viewed from a perspective of mathematical knowledge for teaching we suggest that the classroom dialogue itself provides, probably unintentionally, the means for Maria to clarify the difference between a descriptive statistical and a probability-laden question formulation. The last turn of the last citation is actually the first time when she phrases a probabilistic question in accordance to her contextualization. It appears at the end of the discussion and, we claim that it has not been explicit neither to the students, nor to the teacher from the beginning of the story. Hence, by contextualizing sampling as she did, we claim that also Maria developed her way of making sense of probability through the dialogue and the way she contextualized the question of sampling.

DISCUSSION

By taking this close look at a certain segment of the data gathered from the entire teaching activity, we are not in the position to make claims about the quality of the students' understanding of sampling and the role of sample size for making probability predictions. However, to make claims about the students' actual understanding has not been the issue of the present paper. The aim of the paper was to investigate aspects of teacher competencies in relation to the challenges a teacher might reach when trying to orchestrate the idea of sampling by allocating the discussion to an everyday context. Based on this we can reflect on the enacted object of learning (Marton & Tsui, 2004) in terms of how a teacher use everyday oriented contextualizations to offer students opportunities to discern and learn concepts and ideas of probability theory.

Gal (2005) suggests that the teaching of probability should orient towards everyday situations in order to support the development of probability literacy. Even if we adhere to such a view of teaching, our analysis shows that this might not be an easy enterprise for teachers. Moreover, our analysis suggest that the problem of connecting to everyday situations should not only be considered as an issue of teacher's insufficient mathematical content SMK (Ball et al., 2008). We suggest that, what became crucial to Maria relates to her knowledge of content and teaching. What seem to be crucial for Maria in her attempt to create certain learning opportunities for her students by allocating the teaching to an everyday situational context, is to be aware of what contexts and FPs she is actually communicating. In that sense, our study extends the notion of KCTP to not only being a matter of a teacher possessing a repertoire of forms of representations, techniques and examples (Ball et al. (2008).

Based on the last lines of reasoning, our initial observations indicate that teacher education should challenge teachers to develop a kind of meta-contextual knowledge in order to learn to be explicit about their agenda in addition to developing SMK of probability. If a teacher uses an everyday contextualizing to make sense of probabilistic ideas and concepts, the teachers should be aware of that such a context might involve many items and variables that can obscure the intended ideas and concepts and sometimes even inhibit them from coming to the fore in the teacher's and students' reasoning. This implies the need to challenge teachers to be more aware of the implicit assumptions on which a reasoning rests and how explicit they are communicating the intended object of learning in a whole-class discussions.

Makar and Rubin (2009) emphasize the need of a driving question in the teaching of stochastic. In our analysis we note that the teacher introduces several intentions and questions in the discussion. So, there is no lack of a driving question as such. However, what we consider problematic is that there are several intentions and questions in play simultaneously. We consider the teacher's difficulty to include a clearly defined probability oriented driving question as an important aspect. The students are supposed to become challenged to think about and propose predictions that contain degrees of uncertainty and are based on frequency information. As claimed above, the analysis indicates that most of what Maria says, stay at the level of descriptive statistics. As we can see in the last part of the last quote above, and from what we hear when we listen to the activity in its whole, we consider her to have sufficient subject knowledge to formulate probability oriented questions. Again, we believe that the critical element relates to a meta-level. She have to be aware of what question she mediates to the students and the ways in which the questions she poses direct the communication

There is much left to do to define what a meta-contextual knowledge in probability constitutes of and what its consequences are for the teaching of probability. A more exhaustive teaching experiment, combined with qualitative analysis of interviews or focus group discussions with teachers would help to get further insight to these issues.

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