YOUNG STUDENTS SOLVING CHALLENGING MATHEMATICAL PROBLEMS IN AN INCLUSIVE COMPETITION: ENJOYMENT VIS-À-VIS HELP-SEEKING[1]

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In this paper, we intend to describe the help seeking and enjoyment patterns reported by the participants in an inclusive mathematics competition. Results suggest that they seek help from a variety of sources, mainly family and teachers, and that they enjoy much the problems in the competition; in addition, help seeking and enjoyment seem to be problem-dependent. Some questions for future research are raised.

Key words: mathematics competitions, problem solving, help seeking, enjoyment.

CONTEXT AND RESEARCH QUESTIONS

Inclusive mathematical competitions as motivational environments

Around the world, the number of regional, national and international mathematical competitions has visibly increased, taking many different forms, contents and durations, and targeting considerably wide groups of students in terms of age and mathematical ability levels. Some competitions target highly talented students, as the International Mathematics Olympiad. Others, like the Mathematical Kangaroo contest, have an inclusive nature, welcoming students with various degrees of aptitude for mathematical problem solving and endorsing feelings of fun, joy, pride and mathematics loving.

Experience indicates that the challenging and competitive nature of enrichment projects is associated with students' positive affect towards mathematics and the development of problem solving skills. In fact, empirical research has proved that students' participation in mathematics competitions has an influence on their motivation for learning mathematics, especially at younger ages. Furthermore, both very good students and those who show some difficulties in school mathematics benefit from participating in such activities beyond the classroom (Freiman & Vézina, 2006).

Based on the review of several studies, Freiman and Applebaum (2011) claim that the advantages of mathematical competitions are strongly connected to affective and emotional factors such as: student satisfaction, self-efficacy, and cooperative skills as well as infused love and interest for mathematics. The context of mathematical competitions is therefore reaching much further than the identification of mathematically gifted students. Inclusive competitions are becoming important sites where mathematics is presented as challenging, exciting, accessible to average students, socially and emotionally engaging, and close to the daily aspects of students' lives.

Online competitions: the setting and the questions addressed

SUB12 is a regional online mathematical problem solving competition supported by the University of Algarve. It has been running annually since 2005, and it is intended for 5th and 6th graders (ages 10-12). It is a web-based competition with two distinct phases: the *Qualifying* and the *Final*. The *Qualifying* phase develops entirely at distance through the website and consists of a set of ten problems, each posted every two weeks. Participants are asked to solve each problem, at home, at school, etc., and to submit their answers by e-mail or using the online form available at the webpage. They can use several resources, approach the problems in different ways, and resort to different representations. Participants have total freedom in presenting their solutions (handwritten and scanned, using the computer to create images or diagrams…). The explanation of the solution process is a fundamental requirement for a complete answer. Regardless of the level of mathematical sophistication, all the correct and complete answers are equally valued.

Feedback is always provided to the participants by a team of senior mathematics teachers. Feedback is formative and encouraging, offering suggestions when needed to help overcoming obstacles, or recognizing successful answers. Students are allowed to submit revised solutions as many times as needed within the respective deadline. Help seeking is approved and even encouraged during the *Qualifying* phase given the inclusive nature of the competition.

Usually 10% to 15% of the total initial participants reach the *Final* phase as they have to submit correct and complete answers to, at least, eight of the ten qualifying problems. The *Final* is held at the university campus. Family members and also teachers accompany the participants but the latter compete individually and on-site.

Acknowledging the role of affective variables within inclusive mathematical competitions, we intend to get a picture of the SUB12 participants' enjoyment and help-seeking behaviour in relation to the problems proposed. In particular, we consider the following questions: (1) What is the significance of the help provided by the several partners that participants in SUB12 can resort to during the competition? (What is the quantitative dimension of this support? What is the most salient source of help and to what extent is the presence of external aid constant or variable over time?); (2) How do participants in SUB12 express themselves in relation to their higher or lower enjoyment with the various problems posed? Is it possible to detect situations of greater preference or dislike towards the problems released? and (3) What trends can be identified combining these dimensions?

THEORETICAL PERSPECTIVES

Challenging mathematical problems

In any learning context, performance must be understood in light of a strong bond between affect and cognition and recent theorizations are not reducible to the identification of causal links between affect and cognition. Understanding affect is

now moving beyond the usual way of seeing it as the other side of cognition; instead, affect is viewed as a part of thinking: "Affect influences thinking, just as thinking influences affect. The two interact" (Walshaw & Brown, 2012, p. 186). In fact,

> Affect is as central to understanding the character of educational experiences as are motivation and cognition. Furthermore, affective, motivational and cognitive processes, while they can be separated conceptually and empirically, are interdependent in the ongoing experience of students. (Ainley, 2006, p. 391)

The very notion of challenge reflects how affect must be integrated with the cognitive aspects involved in it. By definition, a challenge presupposes an element of difficulty, raises the need to overcome an obstacle. Barbeau (2009) has thoroughly elaborated on the idea of challenges in mathematics education. His view stresses that mathematical challenges deliberately incite its recipient to attempt a resolution. A good challenge is one for which the individual has the necessary mathematical apparatus but is required to deal with it in innovative ways, thus implying the feeling of being intellectually alive and able to share some of the thrill of devising new approaches as mathematicians usually do. Such mathematical challenges are usually seen by the students as different from the regular problem solving activities in school and, even when perceived as difficult to grapple, the challenges boost feelings of enjoyment (Jones & Simons, 1999, 2000).

The various problems proposed in SUB12 briefly and succinctly describe a context-framed situation, casting a well-defined question, but they are expected to be seen by participants as challenges, which amounts to believing that students feel inwardly compelled to solving them. Therefore we propose a delicate difference between the idea of mathematical problem and the concept of challenging mathematical problem. A *mathematical problem*, usually conceived as a situation from which the initial and the final states are known but the process to move from the first to the last is not immediately available through mathematical techniques and reasoning, has its grounds on the cognitive components of the problem solving activity. In turn, a *challenging mathematical problem* includes a strong affective appeal by involving curiosity, imagination, inventiveness and creativity, therefore resulting in an interesting and enjoyable problem not necessarily easy do deal with or to solve (Freiman, Kadijevich, Kuntz, et al., 2009).

The idea of moderate challenge

Research has stressed the need for balance in the degree of challenging questions (or problems) posed to students and the idea of *moderate challenge* has come to the fore (Turner & Meyer, 2004). The propensity to perform a task seems to decrease in two situations: when one's expectations about the probability of success are very high (too easy task) or very low (too difficult task). The preference for challenges rests on the situations in which the expected success is around 77%.

The environments in which help seeking is regarded as natural are consistent with those that value moderate challenges. Such challenges seem to be ideal for

persuading students to try and encouraging them to explain alternative strategies, evaluate approaches and appreciate multiple possible solutions. Thus, the use of moderate challenges has much to gain when associated with features that are typical of contexts which value challenge. One of such features is viewing help seeking as legitimate and another is pressing for explanations and accountability for thinking. These two aspects are clearly present in SUB12 – not only is help seeking explicitly encouraged as reporting the solution process is required – and they are precisely the two essential categories that may describe challenge supporting practices, according to Turner and Meyer (2004). Interestingly it is reported that students who are given moderate challenges tend to reveal lower avoidance of help seeking.

On the occurrence of students' help seeking and help avoidance

When a participant seeks help, can we assume that the problem was actually seen by the participant as a challenge? If not, why? We may suspect that the degree of difficulty of the problem was probably too high, leading to the need of seeking help. Moreover, the very act of asking for help can compromise the challenging character of the task in the eyes of the participants since the sense of achievement, namely if it is equated with performance demonstration, may not be as full – the credit for having answered well goes not just to the participant but is shared with others.

Help seeking has been researched from the perspective of a behavioural type of self-regulation and has received increasing attention for its role in the learning process. Zusho and Barnett (2011) highlight some of the developments on the conceptualization of help seeking and help avoidance, namely stressing the social connotations of help seeking in tune with the costs involved: being perceived as needy and admitting failure or incapacity to accomplish a task. There is evidence that self-regulated and confident learners are more likely to display adaptive profiles of help seeking, which means looking for instrumental help: where the reasons to find help are the wish to learn and to understand the material, as opposed to a shortcut to get a task completed. In addition, empirical evidence suggests that help avoidance is a consequence of performance expectancy as help seeking is seen as a threat to self-worth and self-efficacy. In fact, low achievers tend to perceive greater threat in help seeking and therefore report higher levels of help avoidance; reversely, students with higher perceptions of cognitive competence show lower levels of help avoidance.

Furthermore, help seeking is adjustable to contextual factors, and patterns of help seeking are consonant with a caring, supportive and exploratory learning environment. This is also related to students' perception of moderate challenge where conditions of support and accountability for understanding are nourished and where a preference for challenging activities goes together with engagement and enjoyment. In such environments, students' preference for solving problems on their own may rise and help seeking becomes closer to seeking clues rather than answers (Zusho & Barnett, 2011).

On the relationship between challenges and affect

The meaning of moderate difficulty is not universal because different people perceive the same task differently. Even the same person can experience different levels of challenge in a certain task depending on having freely decided to engage with it or on having been required to perform it (Schweinle, Meyer & Turner, 2006). Although acknowledging the relativeness of moderate challenge, there are indicators supporting the claim that targeting moderate challenge is a favorable condition to develop positive affect. But other conditions should surround moderate challenge: a social environment that supports enjoyment, self-confidence and value in mathematics – a number of characteristics that SUB12 fulfills, all of which are grounded on the kind of feedback provided, which is a key element, amongst others, that contribute to the inclusive nature of the competition. Inclusiveness is part of the global aim of fostering enjoyment (by easing frustration, giving positive reinforcement, encouraging persistence, valuing cooperation more that competition – as reflected in the opportunity for children to participate in small groups).

> Optimal levels of challenge, coupled with affective and motivational support, can provide contexts most supportive of students' feelings of enjoyment, efficacy, and value in mathematics (Schweinle, Meyer & Turner, 2006, p. 289)

There seems to be a highly interactive relationship between positive affect, challenge, and value attributed to mathematics (and more precisely to mathematical tasks). In particular, we can refer to two kinds of threats to students' perceived ability, both possibly combined: the difficulty of the task and the need to seek for help. The two threats, especially when combined, can generate negative affect, which is observed in our study in terms of participants' level of enjoyment regarding different problems proposed in the course of SUB12.

METHODOLOGY

In this paper, we address two dimensions of 5th graders' participation in SUB12 concerning affect and emotion: help seeking and enjoyment in solving the challenging mathematical problems presented throughout the *Qualifying* phase. Data were collected through the participants' answers to a mini-questionnaire consisting of two multiple-choice questions included in the online form available on the webpage to submit the answer to each of the problems posted. The two items were straightforward and the answers were given by choosing a single option: 1) I solved the problem with the help of: a) Teacher; b) Family; c) Friends; d) SUB12; e) Nobody; and 2) I enjoyed the problem: a) A lot; b) So-so; c) Little.

Answering the questionnaire was mandatory when participants chose to use the online form to send their problem solution. However, the right not to respond was assured – alternatively to using the online form, participants could choose to send their answers directly to the SUB12's address using their personal e-mail. All participants in the competition are required to have an e-mail account.

The number of respondents to the questionnaire is slightly below half the number of participants enrolled and basically corresponds to those who used the online form to submit their answers. It is important to notice that as the competition unfolds the number of participants decreases (by attrition) and so does the number of respondents to the questionnaire (there were 469 replies at the beginning and 151 at the end, as shown in Tables 1 and 2).

Our analysis of the data is guided by the research questions and mainly intends to look for patterns that may help to understand the significance of help seeking and the level of enjoyment reported in each problem, and also possible associations between those. Since the mini-questionnaire was only launched on problem #2, the data consist of answers referring to problems #2 to #10. Our approach is mainly descriptive, based on the number of answers and percentages regarding each option per problem, by looking at such values across the series of problems.

DATA ANALYSIS

A first indication on how participants report on help seeking is quantitatively expressed for each of the nine problems in Table 1. Similarly, Table 2 displays the distribution of answers to the item on enjoyment per problem.

Prob. 2 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
190	42	126	102	9	469
40,5%	9,0%	26,9%	21,7%	1,9%	100,0%
Prob. 3 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
235	48	85	90	5	463
50,8%	10,4%	18,4%	19,4%	1,1%	100,0%
Prob. 4 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
150	31	98	75	6	360
41,7%	8,6%	27,2%	20,8%	1,7%	100,0%
Prob. 5 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
75	27	57	91	3	253
29,6%	10,7%	22,5%	36,0%	1,2%	100,0%
Prob. 6 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
99	19	47	57	4	226
43,8%	8,4%	20,8%	25,2%	1,8%	100,0%
Prob. 7 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
120	11	52	48	5	236
50,8%	4,7%	22,0%	20,3%	2,1%	100,0%
Prob. 8 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
95	23	59	54	1	232
40,9%	9,9%	25,4%	23,3%	0,4%	100,0%
Prob. 9 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
44	20	66	65	0	195
22,6%	10,3%	33,8%	33,3%	0,0%	100,0%
Prob. 10 - Help					
Nobody	Friends	Teacher	Family	Sub12	TOTAL
52	16	38	43	2	151
34,4%	10,6%	25,2%	28,5%	1,3%	100,0%

Table 1: Help seeking in each problem

Prob. 2 - Enjoyment			
A lot	So-so	Little	TOTAL
295	155	19	469
62,9%	33,0%	4,1%	100,00%
Prob. 3 - Enjoyment			
A lot	So-so	Little	TOTAL
340	115	8	463
73,4%	24,8%	1,7%	100,00%
Prob. 4 - Enjoyment			
A lot	So-so	Little	TOTAL
245	97	18	360
68,1%	26,9%	5,0%	100,00%
Prob. 5 - Enjoyment			
A lot	So-so	Little	TOTAL
136	109	8	253
53,8%	43,1%	3,2%	100,00%
Prob. 6 - Enjoyment			
A lot	So-so	Little	TOTAL
147	66	13	226
65,0%	29,2%	5,8%	100,00%
Prob. 7 - Enjoyment			
A lot	So-so	Little	TOTAL
169	55	12	236
71,6%	23,3%	5,1%	100,00%
Prob. 8 - Enjoyment			
A lot	So-so	Little	TOTAL
140	81	11	232
60,3%	34,9%	4,7%	100,00%
Prob. 9 - Enjoyment			
A lot	So-so	Little	TOTAL
114	58	23	195
58,5%	29,7%	11,8%	100,00%
Prob. 10 - Enjoyment			
A lot	So-so	Little	TOTAL
99	51	1	151
65,6%	33,8%	0,7%	100,00%

Table 2: Enjoyment in each problem

In what concerns the distribution of reported help seeking, the data show that it has a visible expression in all problems. Participants in SUB12 widely vary in their abilities to solve mathematical problems and this is an inherent feature of the inclusive nature of the competition. In general, a large percentage of these participants openly declared to have received help (help seeking was only slightly below 50% in problems #3 and #7). It is also important to recall that help seeking is actually encouraged and stimulated by the organization and explicitly stated at the webpage. Thus, fifth graders (the ones entering the competition for the first time) showed to be willing to seek for help in their problem solving activity during the *Qualifying* phase.

The two major sources of help, family members and teachers, reveal similar percentages, although there are pronounced differences in some of the problems (problems that are more related to daily life may become good opportunities for family members to help their children). It may well be that the same teacher is a source of help to a large number of participants – there are some teachers who reported in interviews that they gave support to their students throughout the competition, and so the help of teachers may cover a lot of participants. The third source of help is the participants' friends, although the amount of inputs from friends is clearly smaller. Finally, as students report it, the SUB12 (i.e. the organization of the competition to whom the participants contact by e-mail) is a residual source of help. However, all participants who sent an initial wrong or incomplete answer to each problem received feedback from the organization in order to reformulate their answer, and eventually this resulted in a correct solution. There is a discrepancy between the percentage of participants who claimed to have received help from SUB12 and the number of cases that actually succeeded after receiving feedback from the organization. A question then comes up: is it true that participants only recognize to have received help when they explicitly asked for it? The participants may not perceive the feedback constantly provided by SUB12 as actual help since such feedback is offered without being requested; it arises as a reaction (and is possibly seen as corrective stance rather than a means to help improving the work already done) to the answer sent by the participants.

The case of Luís (L) is illustrative of the referred situation (Figure 1). Luís sent a first answer to problem #5 which was not completely correct and stated that he had help from family and enjoyed more or less the problem. The SUB12 offered him feedback and some hints. He then reformulated his answer and resent it, claiming that nobody had helped him and maintaining the level of enjoyment. SUB12 answered again indicating a faulty answer and giving more feedback. Finally, Luís got the solution right and resent it, confirming having had no help and a more or less enjoyment.

There are few cases in which participants explicitly address the SUB12 asking for help to start solving a problem. In such rare cases, the fact is that participants declare that they had help from SUB12, recognizing to have sought such help.

From:	**L**
Sent:	Saturday, 17 March 2012 16:45:20
To:	Sub12

Q1: Family
Q2: So-so

Answer: (…)

From:	**SUB12 Campeonato**
Sent:	Sunday, 18 March 2012 11:09:00
To:	**L**

It is almost correct. You know that each side of the closed table is 60 cm long. You know that each leaf is 30 cm wide. Thus the open table is 60 +30 +30, which is 120 cm, long. But notice that the cloth falls 10 cm on each side, when the table is open. So you have to calculate the length of the cloth, increasing these 10 cm on each side. And the width of the cloth, what is it?
We wait for your correct answer.

From:	**L**
Sent:	Sunday, 25 March 2012 19:40:52
To:	Sub12

Q1: Nobody
Q2: So-so

Answer: (…)

From:	**SUB12 Campeonato**
Sent:	Sunday, 25 March 2012 20:12:39
To:	**L**

Hi, L!
We received your new answer to problem 5. You have to think better on your resolution
It's not completely correct yet. Note that the table cloth fell 40 cm on each side with the table closed. And the table is square, so how will the cloth be?...
We wait for your correct answer.

From:	**L**
Sent:	Tuesday, 27 March 2012 18:25:19
To:	Sub12

Q1: Nobody
Q2: So-so

Answer: (…)

Figure 1: Sequence of e-mails between Luís and SUB12

In what concerns enjoyment, the overall manifested feeling shows a general positive emotion of participants in facing challenging mathematical problems. However it can be noticed that in problems #5 and #9 the number of answers stating "much enjoyment" is lower. At the same time, for these problems there is an increase in the number of answers indicating "so-so" and "little" enjoyment. These two challenges are, in a sense, the deviants within the category of the enjoyment felt and are also those in which participants report having sought more help (more than 70%).

Given the volume of help sought in problems #5 and #9, we can infer that these problems raised more difficulties than others to the participants. This complexity seems to be associated with less enjoyment in solving these problems. This greater difficulty for participants also seems to indicate that the challenge was higher and therefore such over-challenging problems led to less positive emotions, namely,

lowering the feeling of enjoyment. The greater search for help in these two problems may also have contributed to a lower enjoyment. There may have been participants asking for help on these two problems who did not feel this need in others, which may have contributed to a sense of weakness and thus to lower enjoyment. The data may also suggest that enjoyment may be problem-dependent, which resonates with the way students perceive the value and interest of different tasks. On the other hand these problems relate to mathematical topics in which students typically have difficulties (geometry and fractions), which may also contribute to explain their help seeking behaviour. So, help seeking also appears to be problem-dependent.

DISCUSSION AND CONCLUSIONS

As research suggests help seeking is an important matter in any learning context, being even more relevant within an inclusive mathematical competition. Our data reveal that participants significantly seek for help. Help provided by different sources positively contributes to the success in the competition and students' sense of accomplishment; in addition it positively influences the quantity and diversity of students enrolled. Participants seek help mainly from two sources: teachers and family. This is a sign of a great family involvement alongside with a presence of the competition in the school environment. Further research should follow to better understand how students perceive help seeking according to the different available sources, in particular SUB12. This source of help is especially intriguing since it seems to be recognized as such only when participants specifically ask for it.

In general, participants do enjoy the challenging problems proposed in the competition, and we believe that most of the problems can be considered of moderate challenge. This seems to resonate with prior studies which indicate that the challenging and competitive nature of enrichment projects such as SUB12 is associated with a positive affect towards mathematics and the developing problem solving skills (Kenderov, Rejali, Bussi et al., 2009). The problems where the level of enjoyment lowers are precisely those for which participants sought most help. Furthermore enjoyment decrease and help seeking increase are located in two specific problems: enjoyment and help seeking are problem-dependent. We argue that the degree of challenge of such problems is higher in that they involve mathematical ideas that are typically difficult in school curriculum. Another question can be raised: is the decrease of enjoyment related to a higher degree of difficulty or is it associated with the need to ask for help?

NOTES

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