MATHEMATICS SECURITY AND THE INDIVIDUAL

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This paper complements a report (Charalampous and Rowland, 2012) regarding the feeling of security which mathematicians can draw from mathematics. The focus of this report is the potential of mathematics to empower the individual, offering possibilities both for distinguishing and transcending one's individuality. The research involved 19 *adult mathematicians working in universities and schools in Greece. Semi-structured interviews revealed that the participants distinguished mathematics from other sciences, and valued both the intellectual abilities which it cultivates and its social contributions.*

Key words: security, fear, mathematics, individual

INTRODUCTION

It is generally acknowledged that affect plays an important role in mathematics education. The literature has focused largely on negative responses to mathematics and on constructs such as mathematics anxiety, fear of failure, and mathematics avoidance (Zan, Brown, Evans & Hannula, 2006). However, mathematics may also engender positive experiences. For example, Russell asserts that:

> The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry (Russell, 1919, p. 60).

The topic of positive experiences emerging from engagement with mathematics is under-investigated. People who confess such experiences are usually mathematicians, but it is reasonable to suppose that the causal link moves from their experience to their choice of profession and not vice versa.

This paper continues the discussion on the potential of mathematics to contribute to one's sense of security, as introduced by Charalampous and Rowland (2012). There we explored the concept of security, as it emerges from the relationship of mathematicians to mathematics. We conceptualised security as the absence of existential fears and investigated two pairs of opposing fears: *assimilation* versus *isolation*, and *change* versus *stagnation*. In the previous paper we presented the findings concerning the latter pair, whilst here we focus on the former.

In the next section we draw out conceptions of mathematics that could be appealing to certain individuals in terms of security, and elaborate on our conceptualisation of security for the purposes of this study. We then proceed with an account of our findings from interviews with a sample of mathematics professionals.

THEORETICAL FRAMEWORK

In this section, we frame our investigation in terms of the nature of mathematics itself, and of security as a psychological phenomenon.

The nature of mathematics

A wide range of perspectives about the nature of mathematics has evolved since early Greek civilisation up to the present time (Davis and Hersh, 1980, Friend, 2007). Plato introduced the idea that mathematical objects pre-exist, awaiting human discovery. Early modern philosophy (Descartes, Spinoza, Kant) lent support to this conviction privileging reason over the sense-experience (Hutchins, 1952). Faith in the power of reason culminated in the quest for secure foundations for mathematics, i.e. a system comprising a few axioms and deductive rules from which all mathematics could be deduced. This dream had to be abandoned when Gödel proved that any such system complex enough to include arithmetic is necessarily incomplete. The collapse of the formalist project led to the re-conceptualisation of mathematical practice as a human and unavoidably fallible enterprise (Hume, 1978; Lakatos, 1976). This view of mathematics as negotiable and consensual is emphasised in social constructivism as a philosophy of mathematics (Ernest, 1998). Admittedly, modern epistemologies-ontologies of mathematics seem to place the mathematician on shifting sand, but nevertheless give them agency in an unfolding mathematical story.

Security

Psychology has recognised the need of security in various ways (e.g. Maslow, 1970). In this research, the concept of security is framed in two steps: (i) by reference to dictionary definitions of security as 'freedom from fear or anxiety' (e.g. www.merriam-webster.com); (ii) a typology of fear due to Riemann (1970). Riemann proposed four types of fear which correspond to four types of personal need and are organised into two opposing pairs. The first pair concerns the individual, the second their environment over time. Humans must achieve a balance between their need (a) to exist both as individuals and as part of an entity that transcends their restricted individuality and (b) to lead a life marked both by stability and innovation. The associated fears are (a) fear of *assimilation* [our translation] versus fear of *isolation* and (b) fear of *change* versus fear of *stagnation*. More specifically about the fears which are handled in the paper: fear of assimilation is connected with losing one's individuality, the need to be distinguished from others and define one's self against others; on the other hand, fear of isolation is connected to avoiding loneliness either physically by belonging to a group of people or metaphysically by participating in an ideal 'essence' elevated beyond the human level (e.g. the universe or God).

The following quote illustrates the relevance of security to mathematics education and the potential application of Riemann's framework in order to interpret this

feeling. Here Mendick (2005, p.175) comments on the pleasure that Phil, a young mathematics student, draws from his engagement with mathematics.

> Phil finds a security in mathematics that enables him to construct himself as intellectually mature and as distant from his working-class, minority-ethnic self.

Phil's response can readily be construed as a response to fear of assimilation; through his engagement and success in mathematics, he positions himself as distinct and distinctive, in terms of his intellectual capacity and his distance from his origins.

METHODS

We explored the concept of security with an opportunity sample of 19 Greek adult mathematicians. Nine of them were in university positions and ten were teaching in secondary schools. In particular, among the first group there were five applied mathematicians (Faidra, Paraskevi, Periklis, Themis, and Vasilis) and four theoretical mathematicians (Alvertos, Dimitris, Kleitos, and Sofoklis). Three of the teachers were teaching in lower secondary schools (Marios, Sokratis, and Stamatia), two in upper secondary schools (Nestoras and Avgoustis), and the rest were teaching in a technical upper secondary school (Aris, Eleftheria, Fanis, Loukas, and Thodoris). All the participants had at least an undergraduate degree in mathematics, which is necessary for teaching mathematics beyond the primary level in Greece. Most of them had substantial professional experience (15 years or more); Themis, Vasilis, Sofoklis, Periklis, Faidra and Eleftheria had been in post between 2 and 8 years. The names used in the paper are pseudonyms.

One semi-structured interview was conducted by the first author with each participant. Four issues were considered: the participant's personal history regarding mathematics (e.g. Describe the history of your relationship to mathematics. Is there any memorable incident?); their views about mathematics (e.g. What do you think distinguishes mathematics from the other sciences?); the relevance of mathematics in everyday life (e.g. Would you say that mathematics has helped you deal with your everyday life?); and their feelings about mathematics (e.g. What do you enjoy (or not enjoy) in your involvement with mathematics?). Views about mathematics were included since the history of philosophy of mathematics provides ontological and epistemological links to security, e.g. the mathematical certainty of Platonism. While discussing the participant's relationship with mathematics in a general way, unconscious feelings surfaced, which might have been difficult to access directly (Rubin and Rubin, 1995).

The utterances were coded as relevant to one of Riemann's four types of fear (see above). Several cases of multi-coding arose. Multi-coding *across* pairs is indicative of contribution of the dimension of time (second pair) to one's sense of individuality (first pair); Multi-coding *within* pairs suggests the dialectical relationship between their components, which is built as the individual strives to reconcile the two opposites. For example, secure sense of group membership can liberate a person to

explore and express their individuality. As the data were revisited again and again, a constant comparison method lead to the emergence of broad themes and related sub-themes, associated with each type of fear. The themes and sub-themes associated to *fear of isolation* are listed in Table 1, by way of illustration. Note that themes 1-4 emphasise aspects of mathematics and mathematical activity that have the potential to offer protection against isolation, while themes 5-7 suggest *interconnections* with the other three types of fear and themes 8-10 acknowledge *limitations* in safeguarding against fear of isolation.

Themes	**Sub-themes**
Omnipresence	mode of thought; tool of all sciences; historical continuity
Social contribution	applications; social empowerment
Communication	conciseness and precision; impartiality and pluralism; real life
Teaching	initiation; communication with children; accessibility
Assimilation	initiation and success; talent and omnipresence; tool for/foundation of all sciences
Change	historical continuity and reliability; precision and one reality; omnipresence and connectedness
Stagnation	isolation and diversity; creation and social contribution
Limitations to isolation	teaching; communication
Limitations to change	omnipresence
Limitations to stagnation	historical continuity

Table 1: Fear of isolation: Themes and sub-themes

FINDINGS

As mentioned earlier, this paper reports on our findings relating to the pair *assimilation-isolation*. Our report is restricted to those themes which are directly related to these types of fear. The participant's views are organised under the corresponding themes starting with fear of *assimilation* (relevant codes: abilities, self-confirmation, mathematical self, superiority to other sciences) and continuing with fear of *isolation* (relevant codings: omnipresence, social contributions, communication, teaching).

Fear of assimilation

First, we report participants' views which indicate how mathematics gave them a sense of being different and distinct individuals. Mathematics enhanced the

participants' feelings of uniqueness and self-awareness, and protected them against fear of *assimilation*.

Abilities

The participants perceived the mathematical mode of thought as superior to any other kind. For example:

> The mind of the humanities stops; the mathematical mind is unsatisfied if the answer remains concealed, it goes on a discovering spree (Stamatia).

This mode implied the possession of above-average intellectual abilities which had been further cultivated by mathematics and were applicable to everyday life.

> the ability to think abstractly requires an above-average capacity (Themis)
>
> I enjoy [doing mathematics] because I feel that is good for my brain . . . it makes me more clever (Faidra)
>
> mathematics makes you think more; you think more quickly, more easily; you handle some problems in a more correct way (Vasilis)
>
> the way the [mathematician's] mind functions, is rarely restricted to mathematics (Sokratis).

These views suggest that mathematicians enjoy admirable and respectable abilities which can boost one's self-esteem.

> I have observed that people who in my opinion have considerable potential, have a good relationship with mathematics (Themis).

Self-confirmation

Successful display of the aforementioned intellectual capabilities bestows a feeling of self-confirmation intensified by the perception that mathematics is difficult. For example:

> [solving a problem] is as if you put test yourself to the test and [the 'examiners'] say to you: 'You passed' (Avgoustis)
>
> Sometimes you may think that you've found an original solution, and even though many others may have found it before you, you secretly feel proud; for a moment, you believe that you are the only one that has found [it] (Marios)
>
> [Mathematics] wasn't like history. It presented a certain degree of difficulty; and probably, I felt more successful when I did well in mathematics, while I didn't think it was as important when I did well in history (Faidra).

Some participants asserted that mathematics had increased their self-confidence; mathematics was regarded as a shield against mistakes.

> [Mathematics] has made me calm, confident in expressing my opinion without the need to necessarily persuade others (Aris)

> You feel confident about the quality of your capabilities . . . I believe that [mathematics] makes you set more and more difficult goals . . . Mathematics protects you against falling prey to lies . . . it saves you from traps (Stamatia)

> Mathematicians have confidence in their opinions; it stems from the fact that [in mathematics] you can check the actions you take, see if they are verified (Avgoustis).

Solving problems, either in mathematics or in everyday life, provides a limitless source of self-confirmatory opportunities. Repetitive positive experiences of that kind lead to confidence in one's ability to manage problematic situations.

Mathematical self

Some participants integrated mathematics with their identity. On the one hand, they traced their involvement with mathematics to personal traits and on the other hand, they recognised that this involvement had led to further cultivation of these traits. For example:

> One reason, why I like mathematics, is its consistency and its rationality; if they suit to one's character, then one is attracted to mathematics (Nestoras)

> Mathematical thought is characterised by strictness, precision, methodicalness and these affect your character as a whole and consequently your everyday life (Thodoris)

> I like [mathematics] because it is in my nature to be rational and I like to do things systematically (Marios)

> It is in my DNA (Eleftheria).

Most of them had entered the mathematical adventure in an early age.

> I knew how to count before I went to school; I also knew about fractions. At preschool age, my father asked me how I could share two apples between three children. I couldn't think of a way, and I answered embarrassedly: "I don't know". Then my father asked me if I knew how to share one apple between three children and I answered: "yes, each child will take 1/3". "Then why can't you share two?" asked my father and I realised what the answer was" (Alvertos).

Mathematics was a means to self-fulfilment and a valuable companion on the journey of self-awareness.

> It is a work of art; you don't need anything more, you feel content (Dimitris)

> Look at the person who manages to discover something after considerable effort. He exclaims 'Eureka' . . . he feels happy; he feels complete because he has used his intellect (Stamatia)

> Mathematics empowers people; it is a great tool, which can help humans reflect on their actions . . . it also offers stimuli for introspection. For instance, nowadays the mostly-discussed issue is the financial crisis and mathematics is connected to the economy (Nestoras)

> Mathematics teaches you how to think rationally, how to order assumptions and draw the best possible inferences. Therefore, up to an extent, it bestows power; [it bestows] the ability of self-evaluation and self-criticism (Fanis)
>
> I regard mathematic[al knowledge] as an important acquisition. Doing mathematics is an intellectual activity which leads you to maturation (Nestoras).

These views imply that mathematical skills play a crucial role in the process of shaping a secure identity. Consequently, they are regarded as an indispensable part of this identity.

Superiority to other sciences

Finally, mathematics was believed to occupy a distinguished position in the constellation of sciences: it is independent, it is at the cutting edge of research and it is the foundation of all sciences.

> Mathematics has an admirable trait; it is self-contained in the sense that it is not required to know another science in order to understand it (Themis)
>
> Research in mathematics continues; [it] precedes [research in] other sciences and it causes admiration (Fanis)
>
> [mathematics] is the basis of all sciences; it is behind anything you can imagine. Even philologists say that syntax is the mathematics of language" (Marios).

These views indicate ways in which mathematicians experience and benefit from the importance and high status of mathematics. All in all, doing mathematics was connected to their sense of worth and being valued.

Fear of isolation

Here we report the participants' views which indicate how mathematics enabled them to feel a part of a greater whole. Mathematics enabled the participants to bridge the gap between themselves as individuals and the external world, protecting them against fear of *isolation*.

Omnipresence

The participants believed that mathematics transcends the human being, both statically and dynamically, thus allowing mathematicians to be part of something greater. First, mathematics is all around us, either as the language which discloses the secrets of the universe, or as a mode of thought to tackle problems.

> but no matter if it is music, or a building, or the forces we realise around us, or anything, everything is linked to mathematics (Aris)
>
> Mathematics functions without being seen . . . It is the language of nature; there is not a different world of mathematics (Fanis)
>
> the mathematical mode of thought is involved in all sectors of life (Thodoris).

Second, mathematics is an adventure of the human intellect throughout history.

> when you learn a theorem, it is as if you communicate with the person that proved it (Avgoustis)
>
> when I teach . . . I try to follow the route that the human intellect has followed (Dimitris)
>
> mathematics is not isolated and if you approach it enough you can see its associations with other sectors of life and you can understand that mathematics is part of the civilisation (Nestoras).

Social contribution

The participants concluded that since mathematics is everywhere mathematicians are useful. They claimed that mathematical progress could be translated into social progress. Mathematical knowledge was perceived as power, not only at the individual but also at the social level.

> Let's talk about the various technological instruments which are produced. For example, an x-ray computed scanner. What lies behind its function? Differential equations do. They are behind many other things, for example, your mobile phone (Nestoras)
>
> Even football involves mathematics . . . when I see the ball moving in a trajectory tangent to the ground I know that the opponent must fall in order to intercept it (Aris)
>
> through mathematics we try to interpret what is going on around us and express it in a language that we understand (Sokratis)
>
> if you know a field well, you can promote scientific knowledge [in this field], contribute something, and this can definitely be translated as power (Periklis).

Communication

> I'm attracted to mathematics by its concise, symbolic language . . . All around the world, mathematical language has the same meaning for mathematicians (Fanis).

Some participants claimed that use of the precise mathematical language cultivated communicational skills.

> Mathematics influences the way I think and the way I organize myself; it has made me able to express myself precisely, examine many perspectives simultaneously; in that sense it has influenced my life since communication is facilitated and misunderstandings are avoided (Themis)
>
> I came to love mathematics when . . . I realised that mathematical training had helped me in comprehending social situations (Stamatia)
>
> Two persons who understand mathematics communicate with one another much better (Avgoustis).

Mathematics was also regarded as an advocate of impartiality and pluralism.

> Mathematics perceives its nature as international. Discrimination related to colour, height or hair is absent from mathematics; it plays no role in the process of problem solving. If

> one is taught by mathematics, then one learns to be free of prejudice . . . I know that I can learn from a shepherd; mathematics has helped me not to regard him as inferior (Sokratis)
>
> Multiple dimensions . . . help you see through someone else's eyes . . . [someone else's opinion] can be considered as another dimension (Sokratis).

The participants expressed the opinion that mathematics can bring people together, facilitate communication and solve misunderstandings, creating a better world to live in.

> [Modelling all sciences in mathematics] is desirable . . . afterwards we will be able to code life, to communicate better and avoid failures and mistakes. Most human problems are the result of ignorance. When I am ignorant I make mistakes and the more ignorant I am the more ready I am to deify something (Loukas).

Teaching

Teaching is not restricted to mathematics. It is connected to fear of isolation mainly through the opportunities it offers to communicate with young people and to initiate them into a community of experts. Mathematics was believed to offer the additional advantage of being accessible and tangible.

> [e]ven though many claim otherwise, mathematics is tangible; . . . if you demonstrate to a logical human being that mathematics is based on logic, then mathematics becomes tangible (Aris)
>
> there is an infinite field of questions even for the simplest rules, there are many applications and many levels of difficulty (Themis).

After all

> [d]ifficult things don't exist; we name difficult what we don't know, and the less we know it the more we try to make it look impressive (Paraskevi).

CONCLUSION

In this investigation we regarded feelings of security cultivated through engagement with mathematics. Security was conceptualised as relief from fear by means of a framework based on Riemann's (1970) suggestion of four types of fear organised in two opposing pairs. In this paper we reported on the pair *assimilation-isolation*. The findings provide evidence that mathematics can be used as a means to achieve balance between these opposing fears. As many times before in the history of the philosophy of mathematics, the participants endorsed the belief that mathematics has special attributes which set it apart from other sciences. Mathematics is at the same time the leader and the foundation of all sciences; it is the language through which we translate, comprehend and 'conquer' the world (Guillen, 1995). Moreover, the attributes of mathematics empower the individual, who develops a repertoire of intellectual capabilities such as deriving logical inferences, finding applicable solutions or communicating with precision and conciseness. Mathematics was the

tool which allowed the participants to shape a strong and confident identity (offering relief from fear of assimilation) and to connect to the world (mitigating against fear of isolation).

These findings suggest possibilities for further research: both in the direction of other cultures, investigating similarities and differences, and in the direction of younger participants, including mathematics students, to capture the development of feelings of security through the development of one's relationship with mathematics.

REFERENCES

Charalampous, E., & Rowland, T. (2012) The experience of security in mathematics. In T-Y Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, Volume 2*, pp. 115-122. Taipei, Taiwan: PME.

Davis, P. J. & Hersh, R. (1980). *The mathematical experience*. London: Penguin Books.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics.* Albany NY: SUNY Press.

Friend, M. (2007). *Introducing philosophy of mathematics.* Stocksfield: Acumen.

Guillen, M. (1995). *Five equations that changed the world.* New York: Hyperion.

Hume, D. (1978). *A treatise of human nature.* Oxford: Clarendon Press.

Hutchins, M. R. (Ed.). (1952). *Great Books of the Western World (Vols. 31, 42).* Chicago: Encyclopedia Britannica.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery.* Cambridge: Cambridge University Press.

Maslow, A. H. (1970). *Motivation and personality.* New York: Harper and Row.

Mendick, H. (2005). Only connect: Troubling oppositions in gender and mathematics. *International Journal of Inclusive Education, 9(2),* 161-180.

Riemann, F. (1970). *Die Grundformen der Angst. Eine tiefenpsychologische Studie.* München/Basel: Ernst Reinhard.

Rubin, H. J. & Rubin, I. S. (1995). *Qualitative interviewing: The art of hearing data.* London: Sage.

Russell, B. (1919). Mysticism and logic, and other essays. London: Longman.

Zan, R., Brown, L., Evans, J., & Hannula, M.S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics: Affect in Mathematics Education: Exploring Theoretical Frameworks, A PME Special Issue, 63(2),* 113-121.