THE INFLUENCE OF HOW TEACHERS INTERACTIONALLY MANAGE MATHEMATICAL MISTAKES ON THE MATHEMATICS THAT STUDENTS EXPERIENCE.

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Making mistakes is an essential aspect of learning mathematics, both for the teacher and for learners. Teachers are generally encouraged to use students' mistakes as teaching points particularly in order to expose common misconceptions. However, teachers use a wide variety of interactional techniques to avoid directly and explicitly negatively evaluating a student's mistake during whole class interactions. This gives the implicit message that mistakes are embarrassing and problematic (Seedhouse, 2001). So, whilst teachers may explicitly state that making mistakes is part of learning mathematics, interactionally they are saying that they are problems and are to be avoided.

Making mistakes is an integral part of learning mathematics and many recent initiatives have advocated teachers using mistakes in their teaching, particularly to expose frequent misconceptions and mistakes to avoid. However, because of the interactional structure of the classroom, teachers often avoid explicitly negatively evaluating students' mistakes or mitigate their negative evaluations in some way. This avoidance of explicit and direct negative evaluation portrays the interactional impression that mistakes are to be avoided. Hence, the pedagogical messages surrounding the role of mistakes in learning conflict with the interactional messages given by teachers when they handle these mistakes.

In this paper, the structure of sequences of interactions in which a mathematical mistake occurs are described in detail to reveal the conflict between the pedagogical and interactional messages.

METHODOLOGY

This study uses a Conversation Analysis (CA) approach, particularly drawing upon the CA literature and studies on the preference organisation of repair in interactions. Conversation analysis is an unusual methodological approach in the social sciences in that it is not driven by research questions or a priori analysis. Rather, it studies the social organisation of 'talk-in-interaction' (ten Have, 1990) with the aim to explicate the structures that the participants themselves orient to in the interaction. This paper focuses on describing and explicating the interactional strategies teachers and students use when a mathematical mistake occurs in the interaction.

Repair can be defined as the treatment of trouble in speaking, hearing, or understanding (E. A. Schegloff, Jefferson, & Sacks, 1977). Trouble can take the form of a mistake or error, but is also used more broadly to include any difficulties occurring in the interaction (Seedhouse, 1996). In this article we will only be considering the treatment of mathematical mistakes and errors. The repair of trouble consists of three parts: the trouble source; the initiation of the repair; and the outcome of the repair. The person in whose turn the trouble occurs and the person who initiates the repair and the person who performs the repair may or may not be the same person. For example, the trouble could occur in Jane's turn, John could initiate a repair on the trouble in Jane's turn, but it might be Peter who actually performs the repair.

A great deal of research has been conducted into the structure of repair in different interactional contexts, in particular the preference organisation of sequences of interactions that follow a trouble source. The notion of preference is complex and is not used consistently within the CA literature. The term preference as originally used by Harvey Sacks (1995), refers to the structural features of turn organisation in interactions rather than the psychological meaning of preferring one thing over another. However, this distinction is difficult to make. Most authors focus on describing the common features of preferred and dispreferred responses rather than defining preference explicitly. These features include the markedness of responses, the frequency of types of responses and issues relating to face (Sifianou, 2012). Preferred responses are often unmarked, occur frequently and are not face-threatening. Dispreferred responses, on the other hand, are often marked, given hesitantly or delayed in some way, are seen rarely in interaction and can be interpreted as face threatening acts (Emanuel A. Schegloff, 2007). Bilmes (1988) argues that there is an association between these features of responses but does not accept these as defining features. He argues instead that they indicate a reluctance by the speaker to give the response. This leads to many authors discussing how speakers 'avoid' dispreferred responses (Levinson, 1983; Mey, 1993).

McHoul (1990) examined the preference organisation of repair in geography lessons and showed that the number of other-initiated repairs was greater than self-initiated repairs. This contrasts with ordinary conversation where there the frequency of self-initiated self-repairs is far higher (E. A. Schegloff et al., 1977) i.e. the same person in whose turn the trouble occurs initiates and performs the repair. However, even though other-initiated repairs appear more frequently in classroom interaction, self-initiated repairs are still preferred structurally.

When considering preferred and dispreferred responses in this paper, a response given by a student is considered a preferred response if it is treated as such in the turn that follows. Thus a mathematical mistake could be treated as a preferred response if the teacher accepts it without hesitation, or marking his response in some way. Thus, when teachers use mistakes as teaching points, the occurrence of a particular mistake might be the preferred response. Equally a mathematically correct and appropriate response could be dispreferred if the turn that follows treats it as such. In the data from the study used in this paper, there is an example where the teacher has asked for the mean, median and mode of a set of data and a student correctly gives the median and the calculation made to find it, but the teacher treats it as dispreferred and continues to talk about the previous student's calculation of the mean.

A conversation analytic approach focuses on the contextual information that the participants themselves see as relevant. In the interactions presented here, the roles of teacher and student are oriented to in the interactions and the context of the mathematics classroom is clearly relevant. Other contextual information, such as details of the sample such as the gender of the teacher of the nature of the school are not oriented to in the data and thus are not included in this paper. The extracts used in this article are taken from a study of seventeen mathematics lessons involving four teachers from four different schools and students aged between 12 and 14 years. The lessons were video recorded and are naturally occurring. These recording were then transcribed using the Jefferson transcription system (2004), but for ease of reading only some of the details of this transcription are included in this paper. This article focuses only on the structure of repair of students' turns that contain a source of trouble that relates to the mathematics.

HOW MISTAKES ARE HANDLED IN WHOLE-CLASS INTERACTIONS:

When students give a preferred response, the teacher often positively evaluates this response immediately and without any markers or hesitations, using terms such as 'good', 'yes', 'ok' and 'that's right'. In the two examples that follow, the students' responses are correct responses and are treated as preferred by the teacher. The example below is taken from the third lesson recorded with Edward's class.

1	Kieren:	it could be either because like th- the chance is even.
2	Edward:	okay good. …

Edward lesson 3.

Teachers also often repeat the preferred response, which can occur with or without a positive evaluation:

3	Richard:	… ok so um you said that micro means what?
4	Sasha:	um millionth
5	Richard:	a millionth very good. …

Richard lesson 1.

However, when the students' response contains a mathematical mistake, the teacher tends to avoid direct and explicit negative evaluation. It is very rare to see an immediate 'no' to a student's response. Teachers generally use a wide variety of methods to avoid an unmitigated explicit negative evaluation, they are "doing interactional work specifically in order to avoid using unmitigated negative evaluation" (Seedhouse, 2001, p. 355). In other words, there is a dispreference for direct and explicit negative evaluations of students' mistakes.

Bald negative evaluations

In this study, there were only two examples where a mathematical mistake was treated as a preferred response in the turn that follows. These both occur in Tim's lessons:

21	Tim:	… if I did that in one operation, if I did that in one swift move, what am I actually doing
22	Nic:	dividing by five
23	Tim:	no

Simon lesson 4.

In the situation above, Tim has asked what dividing by 3 and then dividing by 2 would be if you did it as one operation. However, the task that the students have been working for the remainder of the interaction focuses on limits of sequences, and in this case what happens if you continue to divide by 3 and then divide by 2. However, the mathematics in this exchange was not central to the task that the students have been working on for the remainder of the interaction, which focuses on limits of sequences. The other occurrence is during an interaction about the probability of picking a particular cup out of a row of ten cups and Tim asks questions about probabilities related to the rolling of two dice. In neither of these exchanges, when incorrect answers resulted in unmarked negative evaluations in the teacher's responses, were the questions directly related to the overarching task. In other words, the mistakes made were not directly relevant to the learning objectives of the tasks within which they occurred.

The methods that teachers use to avoid negatively evaluating a student's mistake include:

Delaying or mitigating an evaluation

10	Sandy:	it's got a plus sign, you've got to like (.) take it off, but if it's like a minus you like (.) add it on
11	Edward:	(0.6) not quite, what is it about this that means that you have to take it away

Edward lesson 1.

In the extract from Edward's lesson there is a negative evaluations but it is delayed as it follows a pause of 0.6 seconds, and is not a bald 'no' but a mitigating 'not quite'. The initiation of repairs can take place over a series of turns, with the teacher offering multiple opportunities for a student to self-repair.

Initiating a repair

6	Tim:	... what fraction of that triangle have I actually shaded. Alex?
7	Alex:	um a half.
8	Tim:	have I shaded a half?
9	Alex:	no

Tim lesson 1.

In this extract from Tim's lesson, Alex has given the answer of a half which is not the answer Tim is looking for and Tim initiates a repair by rephrasing Alex's response as a question and returning the turn to Alex. There is no explicit negative evaluation of Alex's response and Alex is given the opportunity to repair (or alter) his response. By initiating a repair of the student's turn no explicit evaluation of the turn is made but it does indicate that there is some form of trouble with the student's turn and it offers the student the opportunity to self-repair.

There are many different strategies a teacher could use to initiate a repair, including repeating the question, repeating the students' response (part or whole) but phrasing it as a question as in the extract above. Each of these does not explicitly evaluate the student's turn and offers the student the opportunity to self-repair, treating the student's turn as dispreferred. Many of these strategies can also result in other-repairs, where another student or the teacher themselves provides the correct answer.

12	Drew:	is the range a hundred and seven- seventeen
13	Simon:	range a hundred and seventeen. the range is the biggest number take away the smallest number. the biggest number is a hundred and twenty five, the smallest number is eight, a hundred and twenty five take away eight. Ashley.
14	Ashley:	no because the the range is going to be in days absent so it'll be eight

Simon lesson 1.

The teacher could also offer an explanation of why the student has made a mistake without explicitly stating that their response included a mistake.

15	Richard:	… what names would you give to these things that are in the squares. I'm looking for the technical term for them. Chris?
16	Chris:	formulas
17	Richard:	formulas, you could say. formula often has equals in it, doesn't it like the formula for speed equals distance divided by time…

Richard lesson 4.

In the extract from Richard's lesson, Richard accepts the response answers but explains why it is not the response he was looking for before repeating the question.

The teacher could also accept the response given by the student but in revoicing it (O'Connor & Michaels, 1993) supply the correct answer:

18	Simon:	… quarter of a hundred and nine?
19	George:	twenty seven. point
20	Simon:	twenty seven point two five. twenty seven point two five going across. …

Simon lesson 4.

Questioning Strategies

Teachers have a wide variety of strategies that they can use to handle mathematical mistakes whilst avoiding explicit negative evaluation. The teacher could, for example, begin a sequence of simpler questions that enable the students to be 'led' to the appropriate answer, a technique often referred to as 'funneling' (Mason, 2002). This does not mean to say that teachers do not negatively evaluate students' responses, just that when they do they are usually delayed or mitigated in some way, indicating that they the previous turn was dispreferred. Direct, explicit and immediate negative evaluations are very rare.

THE ROLE OF MAKING MISTAKES IN LEARNING MATHEMATICS:

Mistakes and misconceptions have come to be seen as essential in the learning of mathematics. This is both from the perspective of the teacher and the learner. From the teacher's perspective, they can be "a powerful tool to diagnose learning difficulties" (Borasi, 1987, p. 2, p.2) but can also give insight into how learners are thinking about the mathematics. This is particularly important where learners' conceptions of mathematical ideas differ from those "generally accepted versions of the same ideas" (Sfard, 2008, p. 16, p.16).

It is through making mistakes or considering different conceptions that a great deal of mathematics has been developed (Tall, 1990). Burton's (2004) description of professional mathematical behaviour includes making mistakes and using them to learn new things. Borasi (1987) offers examples of how mistakes and errors can be used within a mathematics lesson to initiate enquiry and enhance learning.

Traditionally mistakes and misconceptions were seen as something to avoid, but Piaget's influence and the influence of constructivist theories of learning began to challenge this and studies, such as those by Askew and Wiliam (1995) and Swan (2001) showed greater learning gains when mistakes and misconceptions were confronted and discussed than when they were avoided. From the perspective of the learner, making mistakes or confronting misconceptions and adapting understanding in light of these is fundamental to constructivist principals of learning.

Many authors emphasise the role of the classroom environment in the handling of students' mistakes. The emphasis being placed on creating an environment where students feel safe to make errors and it is a classroom norm to discuss and explore errors (Schleppenbach, Flevares, Sims, & Perry, 2007). Several studies have introduced or developed rules for interaction to establish an environment where learners feel safe making mistakes and discussing them (Mercer & Sams, 2006; Ryan & Williams, 2001). However it is important to note that student errors do not necessarily mean a lack of knowledge or understanding (Mehan, 1980).

O'Connor (2001) explores the decisions teachers need to make about whether they correct a mistake or not and raised some interesting questions about the role of the purpose of the activity and the role of the mistake within the interaction. This article is not concerned with these conscious decisions that teachers must make but instead looks at how the interactional norms of conversation and classroom discourse may influence how students may interpret the teacher's actions, both when a mistake is corrected and when it is ignored.

The conflict

By treating mistakes as dispreferred responses to questions, teachers are interactionally telling students that making mistakes is embarrassing and face-threatening and to be avoided. There is a conflict between the pedagogical message that making mistakes is an essential part of learning mathematics and the interactional message that mistakes are to be avoided (Seedhouse, 2001).

"Teachers are avoiding direct and overt negative evaluation of learners' ...errors with the best intentions in the world, namely to avoid embarrassing and demotivating them. However, in doing so, they are interactionally marking ... errors as embarrassing and problematic." (Seedhouse, 2001, pp. 368-369, pp. 368-9)

So, whilst teachers are not explicitly telling students that making mistakes or errors is embarrassing and problematic, the way that they handle these in whole class interactions implicitly tells the students that they are. The very responses such as partial agreement or mitigation, that may be intended to allow incorrect answers to contribute to learning, are also the ones that give the interactional message that these errors are to be avoided. So, whilst teachers are not explicitly telling students that making mistakes or errors is embarrassing and problematic, the way that they handle these in whole class interactions implicitly tells the students that they are.

Conclusion

The purpose of this paper was to describe the interactional structure of repair in mathematics classrooms when mathematical mistakes are made. This has revealed a conflict between the interactional messages which treat mistakes as something to be avoided and the pedagogical message that mistakes are an essential part of learning. Teachers use a wide range of interactional strategies to avoid baldly negatively evaluating a student's turn, treating mistakes as dispreferred responses, and hence a source of trouble. However, this paper does not suggest that teachers should baldly negatively evaluate students' mistakes more frequently. Further research is needed to explore the influence of varying these interactional and pedagogical messages may have on the learning of mathematics. Indeed, the implications of raising teachers' awareness of this conflict needs exploration.

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