READING STORIES TO WORK ON PROBLEM SOLVING'S SKILLS.

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As a part of a research on "mediation trough stories" in mathematics class, we present a didactic engineering (Artigue, 1988) built on two storybooks about problem solving activity. To characterise proper reading and understanding of mathematic problem, we define a "reading-contract of problem instructions"(based on Brousseau's didactic contract (1998) and Eco's fictional reading contract (1994)). We assume that studying plot and alternative worlds, described in stories about problem solving, leads children to work on their problem solving skills. Our work is backed by the findings of an experiment lead on a fifth grade class (primary school). The storybook approach allows us to point out benefits of mediation through stories in problem solving.

Keywords: Problem-solving; Mathematics mediation; Mediation through storybooks; Reading contract of problems' instructions; Multidisciplinary.

This work is part of research exploring Sciences' mediation through stories [1] lead by a multidisciplinary team from S2HEP laboratory (Sciences & Société: Histoire, Épistémologie et Pratiques). Based upon the heuristic power of stories (Bruner, 2003) this research points out benefits of studying stories in experimental science classes. Following these positive results, we aim to extend mediation through stories to mathematics. In the context of an on-going doctoral PhD in mathematic didactics [2], we are developing a specific work about problem solving in two complementary ways. The first one involves "story-reading" and the second one "story-writing". With this guideline, we try to open a new field of French Mathematics Didactic.

The main goal of this paper is to disclose benefits of mediation through story-reading in problem solving (especially on problem's instruction understanding skills such as the selection of data). So, we focus here on a didactic engineering (Artigue, 1988), designed for 10 to 11 year old children (primary school), involving the study of two French storybooks. We make the hypothesis that the study of chosen storybooks allows children to develop a proper reading and understanding of mathematic problems. More generally, we assume that stories have a heuristic potential that we can rely on to make children work on their problem solving skills (understanding and solving).

We first present educational elements (analysis of curricula and mathematics textbooks) to frame our work. Then, we share items of our theoretical framework about problem solving and mediation through stories. We carry on with the

engineering, starting with the study of the storybooks. At last, we highlight representative exchanges that occurred in the classroom and extracts of the children's work in order to expose the main results of this work.

EDUCATIONAL ELEMENTS AND OBJECTIVES

About problem-solving

According to French and some European curricula [3], problem solving is the central activity in mathematics. It aims at improving reasoning skills and logical abilities to bring meaning to mathematical objects. French curriculum for primary school insists on the fact that problem-solving may "*increase pupils' knowledge, reinforce sense's control and develop rigour and interest in reasoning*" (Bulletin officiel, 2008, p. 23, T.B.A). Indeed, problem solving activity implies devising a plan based on mathematical reasoning. Referring to Polya's (1945) four-phases model of problem-solving, the first step is to understand the problem [4]. In this specific paper, we focus on this initial stage and try to figure out what a "proper reading and understanding" of mathematical problem instructions must be. In fact, we consider problem instructions as specific texts because they are scholar texts written as short stories. So, as Jacobi (1987) emphasises, they requires specific techniques to be read and understand.

At this point, we questioned ourselves about the existence (implicit or explicit) of a specific contract for reading and understanding problem instructions, consequence or not of Brousseau's didactic contract (1998). So, we made an analysis of some French mathematical textbooks in order to see how school problem narratives are drawn up and how problem reading and understanding are taught. Using linguistic tools (Larivaille, 1974; Reuter, 2009), we disclose some limits in problem narratives structures. We point out a lack of complexity in the narrative backing the problem. Indeed, even if they look like short stories, school problems are missing the main characteristic of a story: the disrupting factor. So, there is only one possible question to ask and more problematic, only one way to solve the problem. The path through problem solving is mapped out (Moulin, & Al., 2012). Because of that, there is no need for pupils to build a mathematical reasoning as they should. They can rely on automatic procedures without reading problem instructions.

A typical exemple of stereotypical problem can be show in this way :

During two night walks, a bunny brings 57 carrots back home. During the second night, he brings back 31 carrots. How many carrots did he bring back on the first night ?	To help his friend, a squirrel took 17 hazelnuts from his supplies. He said: "I still have 41 left". How many hazelnuts did he have in his supplies ?

Image : Translation of Maths + CP (2009, p.125)

In those two problems numbers are in the right order to solve the problems. On the first one, you have to do 57 minus 31 and 17 plus 41 for the second one. When you have to do a substraction the first number is bigger than the second one, and when

you need to do an addition it is the opposite. A lot of problems are built using this structure and we can guess that the authors make this choice to help children to solve problems.

But, this kind of structures allows children to go on with an "incidental solving" and prevent them from working on their reading and understanding skills. However, they need to practice on those abilities to be able to solve mathematical problems with a plan devising based on mathematical reasoning. So, they need to embed "proper reading and understanding". Castellani (1995) points out that good command of disciplinary support, as problem narratives, is essential to acquire knowledge of the discipline. Since problem instructions are built as stories, we relied on Eco's fictional reading contract (1994) to build a "reading contract" for problem instructions. We assumed that the two following rules are the most important:

- Knowing that the world described in the instructions is not the real world but an idealized world (to make calculation easier). The Real world is in fact modelled in mathematics school problems.
- Distinguishing in the instructions what belongs to the story's context and what belongs to the data.

About Mathematics Mediation through Stories

We choose an approach based on storybooks in order to lead children to make explicit these rules. Indeed, as we explain in the next section about our theoretical framework, stories have a heuristic potential. Relying on this characteristic, we assume that studying a story about problem solving activity can lead children to question their conceptions of problem solving activity. Our experimental hypothesis in this work is that the confrontation between character's solving activity and the pupils' habits in problem solving might create debates about problem solving in the class. Storybook reading and studying is not an usual approach but it fits in French curricula. Indeed, according to them, it is necessary for children to reflect on texts and documents:

"They have to interpret and build an argument in literary field but also in scientific field. They have to train in order to mobilize knowledge in complex situations to question, to search and to reason. They have to be able to express themselves in oral or written session and communicate in every situations" (Bulletin officiel, 2008, p. 10, T.B.A).

Our main goal with this work is to study characteristics of mediation through storybooks reading. So, we need to identify stories with a mathematical teaching potential (about problem solving activity in our case) and we have to determine didactical and epistemological conditions for mediation through stories in the mathematics class.

THEORETICAL FRAMEWORK

Theoretical elements about mediation through stories

Multidisciplinary with Sciences is usually made between scientific disciplines. Our choice to link a literary work with a mathematical work is due for one part to school French curricula that recommend the language work in each discipline. Furthermore, as we already said it, mediation through stories produced interesting results in experimental sciences' class. For instance, Bruguière & Herault (2007) show how pupils' first conceptions can be questioned through a story's study and Triquet (2007) points out that a narrative's plot can be a powerful tool for knowledge building.

Bruner's writings (about stories' characteristics and stories' educational potential) claim that stories bring us to question ourselves on our own habits (Bruner, 2003, 2008). Additionally, Tauveron support the hypothesis that understanding implicit data in a story involves a cognitive and cultural work (Tauveron, 2003). The common point linking this works is that fictional stories bring alternative worlds in our environment. Those worlds can lead the reader to have some "thought experiment" and then to question his own relation to objects [5] (Triquet & Bruguière, 2010). In a mathematical story's context, the "thought experiment" and the questioning must be about mathematical objects. Relying on this context, we made the following hypothesis:

- If alternative worlds and implicit data of the story are linked with mathematics and problem solving activity, the reader has to begin a mathematical work in order to understand these elements.
- Studying storybooks with a mathematical context and a plot about problem solving activity leads children to question and then to improve their problem solving skills.

Mathematics didactic elements

In order to test these two hypotheses, we build an experiment based on the "didactical engineering" methodology (Artigue, 1988). This kind of methodology relies on an internal validation. As we did in the previous subsection, you have to determine some hypotheses. Then you build a plan in order to test this hypothesis. The validation is made by the confrontation between an *a priori* analysis of the plan (build an epistemological, didactic and/or cognitive study of tasks) and an *a posteriori* analysis (build on observations and pupils productions).

We also rely on Brousseau's Theory of Didactical Situations (1998) and especially the concepts of environment, action phase, formulation phase and validation phase to build this engineering. For this particular work and hypotheses, our engineering consists in a plan of four sessions based upon two storybooks for children. We present those books in the next section but we must already say that the two plots rely on mathematical problem solving and that the characters' ways to solve it have an interrogative potential.

DIDACTIC ENGINEERING

Through the first part of this article we can come up with the general hypothesis that story-reading can be a way to make children improve their problem solving skills. In order to test this hypothesis, we build a didactic engineering (Artigue, 1988) involving a work on a tale named Le Problème (Aymé, 2002, text from 1944) and a play also named Le Problème (Lamblin, 2000).

Storybooks' presentation

As we said previously, we need storybooks with a heuristic potential about problem solving. As shown by their titles, *the problem*, both stories are built on mathematical problems in a classroom [5]. Into them, children characters have to solve a mathematical problem given by their teacher. This is a necessary characteristic about the plots. Indeed, the two of them rely on the mix-up between reality (character's real world) and the mathematical world (brought by problems' instructions). In fact, the childish characters merge some elements of the problem with their equivalents in the reality and propose an unusual problem solving method. In Ayme's tale, the characters go to a forest to count trees because their problem begins with "the woods of the town" and ends with "how many trees are there in the woods of the town". In Lamblin's play, the fact that the problem instruction included the words "my daddy" leads children to convert all elements in the instructions in order to make them identical to their reality.

The authors of the two stories used the possible mix-up between the sense and the signification of some words. This characteristic allows a work on the explanation of "game of languages", and this explanation can be productive of knowledge (Durand-Guerrier, Herault, & Tisseron, 2006). Because of this mix-up, characters make confusion between data and context elements in the instructions. Then rules of proper reading, as we defined them, are broken. In our engineering we want to make children work on these ruptures. Indeed, some elements, linked to mathematics and problem solving, explaining these ruptures remain hidden in the stories. So relying on Tauveron's (2003) work, we can assume that working on these mix-up will bring children to improve their skills in problem solving activity as we want them to.

Research questions

We lead children to study the characters' problem solving activity in two ways: in relation to the plot and also in relation to the problem instructions. So, we came up with two specific hypotheses for this engineering:

- H1: Studying alternative worlds, linked to the mix up between the real and the mathematical world and working on implicit elements linked to the characters' reasoning, imply a mathematical work about problem solving activity.
- H2: Fiction and "games of language" allow a questioning on the nature of the different words included in the instructions and the referential function of words in mathematical problems' instructions.

By going deeper in the analysis of the plot children might link their problem solving activity to the way used by the characters to work on mathematical problems.

Tasks and *a priori analysis*

We focused on two activities, each one linked to a rule of our "reading contract for word problem instructions" presented in our objectives about problem solving.

The first activity relies on the study of the disrupting factors which are the same in both stories: the mix up between the real word and the mathematical word. In Ayme's tale it occurs when a character suggests going into the forest to count the trees. In Lamblin's play it happens when one child asks his teacher if the problem is about a random dad or about his own dad. We can assume that studying characters' reasoning and results can bring children to build their own judgment about the links between real world and its modelling in mathematic problem instructions. We suggest one set of three tasks in order to enable debates in the classroom. Our hypothesis is that their arguments will evolve with the analysis of the two stories. So, each debate is preceded by individual work with stories' comprehension and analysis questions. As we said previously, our aim is to study the children's knowledge development and we will rely on their arguments' evolution to analyse it. In the following lines we present the main task preceding each debate and the main question.

First debate: What can you say about the characters' methodology to solve the problem? What can you say about their result? We wanted children to build their own judgment about the result and the methodology engaged and then to be able to argue about criticisms that can be addressed to the characters.
Second debate: What is the characters' reasoning (which is implicit in the story and relies on the links between real world and mathematical world)? We want children to debate about the relevance of this reasoning.
Third debate: After reading Lamblin's play, what can you say about the character's attitude (which is to make problem's instruction look like his real world)?

For the second rule of our "reading contract", we wanted children to work on the referential function of the words in the problem's instructions. For them the goal of this set of activities is to identify data and context elements in some instructions and to learn their function in problem solving activity. We assume that studying characters' points of view, which rely on opposite attitudes toward the words included in the problem instruction, can bring children to understand these two notions. In order to test this hypothesis, we ask children to select data and context elements in Ayme's problem at three different times and we study their selection's evolution.

Time 1 with Ayme's tale: After reading Ayme's tale pupils have to select in the instruction what they would use in order to solve the problem. Then, after solving it they have to select the elements they used.
Time 2 with Lamblin's play: Pupils have to study how the childish characters alter the problem instructions and the consequences of those changes in the problem

solving. Each change in the instruction brings a new word problem. We ask pupils to solve all of these problems in order to see solving evolutions (when changes concern data the solving is altered whereas when it's about context elements the solving is identical). The aim for pupils is to identify the different natures of the words: data and context elements.
Then, we ask them to determine in Ayme's problem's instruction what belong to data and what belong to context elements.
Time 3: We ask children to rewrite the problem's instruction to work on data selection and context elements.

Methodology of data collection

We worked with a 10-11 years old class of 28 pupils during four sessions of 50 minutes each. The tasks we just present are spread in four questionnaires (one per session). In this way we can study the evolution of children's answers through the study of the stories' plots. The analysis of the questionnaires complemented by the audio recording of the four sessions allows us to point out some relevant characteristics and to support our hypothesis about mediation through story-reading.

RESULTS

In this section we want to highlight some results showing how mediation, through these two storybooks, operates in the class. As we said in the previous section we focused our analysis on two sets of tasks each one about one hypothesis (H1 and H2).

About the first hypothesis implying alternative worlds (H1)

Our first hypothesis is about the links between the real and the mathematical world. In the two stories alternative ways of solving and reasoning are brought up. We assume that the confrontation between the character's solving and the children's habits will create debates about problem solving in the class.

Indeed, during the debate session, one of the topics discussed was about the difference between "real problem" (about a real situation), "realistic problem" (inspired from a real situation) and "fictive problem" (about fictional elements)". Children in the class established a distinction between these three problem's types. To go further with the first rule of our "reading contract", concerning links between reality and mathematical world, we can also say that children reflect on how the real world is included in word problem instructions. Between the two first debates, pupils' criticisms against the characters' methodology evolved in a very interesting way. They included modelling and mathematical characteristics in their arguments. For instance, we put some significant evolution in the table underneath.

Before the first debate	Before the second debate
- The result is "wrong because they counted the real trees of the forest".	- Problem's woods are not supposed to be realistic. By counting the real trees instead of doing a calculation, characters made a mistake.

- The result is "right because they were careful when the counted and other characters found the same result". - The methodology is "wrong because a problem is unreal". - The methodology is "wrong because you have to do calculations in order to solve a problem".	- A problem is not real (…) it is precise; in each part you know exactly how many trees there are. But in real forests there is not always the same amount of trees in each part. - The methodology is incorrect, there are too many places for mistakes because their forest is not the same as the one in the problem. They spend all day counting and doing calculation is faster.

Table 1: Evolution of pupil's criticisms between the two debates

This evolution is due to the study of the tale. Pupils criticized the modelling of the world into the mathematical world. This shows that children realized that problem instructions are not identical to the reality. The mathematical world is idealized, simplified and overall very regular. They also were able to reflect critically on the other storybook, reinvesting their knowledge about links between mathematics and reality in mathematics problems during the last session.

About the second hypothesis implying fiction and games of language (H2)

Concerning the second rule of our "reading contract", which implies data's and context elements' characterization, we can also say that the study of these two books made pupils understand these concepts.

Indeed, during time 1, pupils were unable to make a select data in the instructions even after they solved the problem. 21 pupils (over 26) were unable to make a difference between data and context in Ayme's problem. During the engineering, the confrontation, between characters' points of view and the study of the evolutions linked to the problem's instructions changes in Lamblin's story, helped children to get the difference between data and context elements. For instance, we translate underneath an exchange that happened during time 2 about context elements:

Student: At the beginning they are complaining that … on daddy is strawberry intolerant so they change it for apples.

(…) *Work about problem solving with apple instead of strawberry in the instructions.*

Tutor: Ok …so, does it change something to solve the problem?

Class: No.

Tutor: Why? Why do we change?

Student 1: Taste! Flavour!

Student 2: Context!

Tutor: Yes, this is context element. It's like the presentation of the data.

Other exchanges involving the same knowledge occurred during that time. After this work (time 2), almost all children were able to distinguish data from context in Ayme's problem (only 6 of them made a wrong selection). They also succeeded in rewriting activity and made context and data changes as they were asked.

CONCLUSION AND PERSPECTIVES

The work conducted in class confirms our hypotheses. The study of the two storybooks helped children to establish some rules of proper reading of mathematics problem instructions. Pupils' answers show that questioning the plot, understanding the stories' problematic and comparing characters' points of views allow children to work on their problem solving skills. As we assumed, mediation through stories reading seems to be efficient in mathematic class.

As we said in the introduction of this article, we are also working on mediation through story writing. We assume that stages in reasoning-making can interact with stages in story-writing. We link the action time, the formulation time and the validation time described in Brousseau's Theory of Didactical Situations (1998) with story-writing seen as an action description (action and validation), or an action anticipation (formulation and validation).

Regarding (Scardamalia & Bereiter, 1987, 1998), we can make the hypothesis that building a story including mathematical elements will lead children to build a reasoning linked to this story. Indeed, they point out that interaction between the rhetorical space (within which the story is built) and the problem space (here in the field of mathematics) leads the writer to improve his knowledge and skills by a process of knowledge transforming. We will rely on this framework to carry on our work about mediation through stories.

NOTES

[1] For a general survey: ASTER n°44 – Sciences et Récits.

[2] PhD Marianne Moulin under the responsibility of Eric Triquet et Virgine Deloustal-Jorrand (S2HEP – University Claude Bernard Lyon 1).

[3] School curricula France (Bulletin officiel, 2008); England (Department for Education, 2012).

[4] Polya (1945) build a four-phase model of problem-solving: understanding the problem, devising a plan, carrying out the plan and looking back

[5] Ayme's problem (T.B.A): The woods of the town are 16 hectare sized. Knowing that each are is planted with 3 oaks, 2 beeches and 1 birch, how many trees are there in the woods of the town; Lamblin's problem (T.B.A): My daddy by a strawberry pie and share it in 4 slices. Knowing that the pie's weight is 800g, how heavy is each slice?

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