FACTORS AFFECTING THE ESTABLISHMENT OF SOCIAL AND SOCIOMATHEMATICAL NORMS

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The present paper initially examines the concept of norm as it has been implemented in mathematics education. A sample excerpt is then used to demonstrate that the analysis of norm establishment would be better accompanied by the consideration of the face-saving needs of the participants. The relevant sociological and linguistic theories are also presented in order to provide the background for that claim.

INTRODUCTION

One of the main current trends in mathematics education research puts the emphasis on collaborative learning environments. In line with sociocultural views on learning been directly related to the societal contexts, numerous studies have been made. It is noteworthy, however, that there are voices that call for attention in the mere acceptance of any teaching/research approach, just because it belongs to the mentioned trend: "the current obsession in mathematics education with group work and discussion is not, and cannot be, a panacea" (Mason, 2004/2008, p. 1). The present paper is the product of the author's interest in the interactional processes that constitute or assist the establishment of taken-as-shared mathematical meanings (Cobb, Wood, Yackel, & McNeal, 1992). In the relevant literature a number of theoretical constructs have been used in order to assist the researchers to describe and interpret the observed phenomena. Among these, I am mostly interested in the concepts of the social and sociomathematical norms as they have been introduced for mathematics education by Paul Cobb and his colleagues (e.g. Cobb & Yackel, 1996; Yackel & Cobb, 1996). These concepts have been used quite extensively in studies that vary from classroom interactions (McClain & Cobb, 2001) to collaborative student pairs (Tatsis & Koleza, 2008) and from kindergarten (Tatsis, Skoumpourdi, & Kafoussi, 2008) to university (Yackel & Rasmussen, 2002). My aim here is to examine whether these concepts suffice to account for the establishment of mathematical knowledge. The answer to this question may be of interest to the researcher or the teacher who wants to interpret the participants' actions while they interact in a mathematical environment. I have to stress though that I do not aim to offer a novel approach in interpreting interactions; I merely wish to examine the complementarity of particular existing theories.

SOCIAL AND SOCIOMATHEMATICAL NORMS

The concept of norm is derived from the broader notion of *prescriptions* which are: "behaviours that indicate that other behaviours should (or ought to) be engaged in. Prescriptions may be specified further as demands or norms, depending upon whether they are overt or covert, respectively" (Biddle & Thomas, 1966a, p. 103). A similar concept to that of the norm is the *obligation* (Voigt, 1994) which connects the various routines of the classroom and regulate the students' and the teacher's actions. Different typologies for the norms have been proposed by sociologists, each based on different criteria; Morris (1966) groups the various criteria into four sets:

(a) distribution of the norm (extent of: knowledge, acceptance and application of the norm),

(b) mode of enforcement of the norm (reward-punishment, severity of sanction, enforcing agency, extent of enforcement, source of authority, degree of internalization by objects),

(c) transmission of the norm (socialization process, degree of reinforcement by subjects),

(d) conformity to the norm (amount of conformity attempted by objects, amount of deviance by objects, kind of deviance).

The above typology was not created to be implemented as a whole; it is rather the researcher's decision on which criteria to use. For example, the distribution of the norm is related to, among others, the extent of knowledge of the norm by subjects (those who set the norm) and by objects (those to whom the norm applies). Thus, in the case of (mathematics) education, a researcher who is interested in the distribution of a norm within a classroom may focus on the objects (students), while one who is interested in the distribution of a norm within the mathematicians' community may focus on the subjects. Yackel and Cobb (1996), while observing an 'inquiry-oriented' mathematics classroom have made another distinction, between social and sociomathematical norms. The former are related to the general structure of classroom activity and some examples are: "explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations in which a conflict in interpretations or solutions had become apparent" (Cobb & Yackel, 1996, p. 178). The latter are specific to mathematical activity and relate to which contribution counts as "a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (Cobb & Yackel, 1996, p. 179). The authors relate the process of establishing norms to the process of constructing mathematical beliefs and values, thus attempting to connect a sociological with a psychological approach. Similar to the concept of norm is that of the *metadiscursive rules* "that is, mostly tacit navigational principles that seem to underlie any discursive decision of the interlocutors" (Sfard, 2002, p. 324). Although Sfard considers norms as a sub-case of rules[1], she offers some useful (for the teacher and the researcher) characteristics of the metadiscursive rules:

> … metadiscursive rules may *evolve* over time (as opposed to the object-level rules of mathematics, which, once formulated, remain more or less immutable). Metarules are also made distinct by being mainly *tacit*, and by being perceived as *normative* and value-laden whenever made explicit. Finally, metarules are *constraining* rather than

> deterministic and are *contingent* rather than necessary. (Sfard, 2008, p. 202, emphasis by the author)

Finally, one could mention the notion of the *didactical contract* introduced by Brousseau (1997), which refers to specific habits of the students that are expected by the teacher and vice-versa. The responsibility of knowledge construction is seen as shared between the teacher and the students and the didactical contract assists that process. However, a familiar didactical contract can also create problems for the students, especially when they enter a situation where the contract changes considerably, e.g. by entering the university.

ROLE THEORY AND THE NOTION OF FACE

All the approaches mentioned in the previous section are to some extent related to a sociological theory. What is common in them (and in other approaches of the same tradition) is the focus on the interactional aspects of the activities that take place in educational settings. The human agent (teacher, student or researcher) is present, but the actions of these agents are usually analysed under the lens of the mathematical activity, e.g. examining the changes in students' knowledge or beliefs about mathematics. What seems to be missing is a consideration of the fact that all participants in any interaction can be viewed as some kind of actors, and sometimes the whole 'scene' is set before them. People's behaviours, seen under this interpretive model, form the research ground for *role theory* as set by Biddle and Thomas (1966b) and exemplified by Goffmann (1971, 1972). The central concepts in that theory are *face* and *role performance*. Face is defined as "the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact" (Goffman, 1972, p. 5) and can be further categorised into positive and negative: positive face is related to a person's need for social approval, whereas negative face is related to a person's need for freedom of action (Brown & Levinson, 1987). The participants not only have these needs, but recognise that others have them too; moreover, they recognise that the satisfaction of their own face needs is, in part, achieved by the acknowledgement of those of others. All these affect the role performance which is defined as "all the activity of a given participant on a given occasion which serves to influence in any way any of the other participants" (Goffman, 1971, p.26). For example, when a student enters the mathematics classroom s/he is primarily concerned about maintaining face; in order to achieve that s/he has to deploy various linguistic and non-linguistic strategies. The same is the case for the teacher. Each communicational move may be initially characterised as face saving or face threatening for the speaker and/or the listener. A request or a question from the teacher for example, is considered a face threatening act; a student's wrong answer on the other hand, may or may not be face-damaging, depending on the classroom's established norms. The effect of the consideration of face can be seen in the use of vague language (Rowland, 2000), which in turn affects the 'quality' of mathematical propositions, thus the learning of mathematics itself. But why would someone use vague language, especially in the mathematics classroom? A comprehensive list is given by Channell (1985, 1990, 1994, as quoted in Rowland, 2000, p. 67):

- giving the right amount of information;
- deliberately withholding information;
- saying what you don't know how to say;
- covering for lack of specific information;
- acknowledging and achieving an informal atmosphere;
- expressing uncertainty;
- downgrading the importance of something as to highlight something else;
- expressing politeness, especially deference;
- protecting oneself against making mistakes.

The above list provides, among others, direct links with the concept of face (positive and negative) and its protection against possible threats. It also stresses the point that I made at the beginning of the section that participants in any interaction, including students in the mathematics classrooms, may be viewed as persons with aims similar to those of any other person. Sometimes, for example, a student might feel the need to not convey the information requested by the teacher; and this might be due to some (inter)personal reasons, related to saving face. In the next section I will present an example of the implementation of these theories; my aim is to demonstrate their compatibility with the concepts of social and sociomathematical norm.

AN EXAMPLE

Mathematical concepts are (or should be) clearly defined; the same is the case with the rules of logic, which assist the mathematicians in expressing conjectures, proving theorems and generally conducting research in pure mathematics. The rules related to these processes can be referred to as object-level rules (Sfard, 2008). However, the interactions taking place in a mathematics classroom or in any other research setting are governed by metarules and norms, whose main characteristics, as they have been already described, are quite far from being fixed. I believe that what provides these norms with their flexibility is the fact that human agents are involved in their establishment. The needs of these agents may evolve over time, thus the norms have to be renegotiated or even replaced. For example, turn-taking – or, more generally, who has the right to speak – may be of high importance in the first years of schooling, but, as students mature, the norm of the quality of one's contribution should be valued. This could be seen as an example of a social norm (on turn-taking) being transformed into a sociomathematical norm.

Two issues need to be addressed at this point. The first is that an established norm does not necessarily have to be a desirable one; in other words, a norm might hinder – instead of assist – mathematics learning. A characteristic example is the ‘mathematical differentiation norm’ (Tatsis & Koleza, 2008) according to which mathematics is comprised of distinct, non-overlapping fields, such as algebra and geometry; there is also a distinction between mathematical and practical solutions. In the following excerpt two female student teachers have just found the answer to the following problem: *Figure 1 shows a triangle in which three lines are drawn to one or the other of the opposing sides from each of two vertices. This divides the triangle into 16 non-overlapping sections. If 14 lines are drawn in the same way, how many non-overlapping sections will the triangle have?* The interaction takes place in a laboratory setting and the researcher has taken a passive stance throughout the interaction by choosing not to intervene or assist the student teachers.

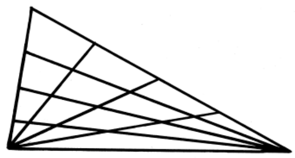

Figure 1: Number of non-overlapping sections

134 Student A: Okay, if we draw three segments we have four triangles. But this is practical. You may say that in three, if you draw three line segments from the one side and also from the other, it created four line segments, four triangles, with four sections each. Right? So, in total the overlapping will be four times four, 16. If you draw 14 line segments you will create 15 triangles with 15 overlapping sections. So, okay, does this have 100% proof? And if someone tells you: I with 60? It will… 61 triangles that will be created, with 61 non-overlapping sections. So, indeed it will be 16 here, so indeed it’ll be 225 here, so indeed 3600 and so on.

135 Student B: Good, if we justify about 15, let the other one with the 60 bother for that.

136 Student A: Do we have an eraser or something?

137 Student B: Why, does it necessarily have to be like that?

138 Student A: What?

139 Student B: A formula?

140 Student A: We are not talking mathematically, that’s all, okay? And what I say is correct, to say: since with three is the immediate larger in 16…

The two students agree to write down the solution, but when student B starts writing, the following exchange takes place:

149 Student A: It can’t be. Something else must be happening here. You keep writing, keep writing.

150 Student B: What was Thales’s Theorem about? It doesn’t fit.

151 Student A: For parallels, angles… let it go. It can’t have theorems, here it’s a practical issue.

152 Student B: If it’s practical, the way found is correct.

The above excerpt can be seen under many different lenses, but for the purpose of the present paper I focus only on the norms which are evident from the students' contributions. The most apparent is the norm of cooperation, according to which the students are expected to work together to solve the given problem; this is evident by the use of the first plural form 'we' in most utterances. What is also evident throughout this brief excerpt is the students' need to 'mathematise' either by creating a formula or by proving their conjecture that the number of non-overlapping sections equals to the number of lines drawn increased by 1 in the power of two. These are manifestations of the norm related to the mathematical efficiency of a solution as described by Yackel and Cobb (1996). Finally, we may observe the norm of mathematical differentiation mentioned before, especially in [134], [140] and [151]-[152]. At the same time, there are some utterances that can't be fully interpreted by the concepts of norms; a characteristic case is [150] where student B asks a question which she immediately withdraws. One may wonder why she did not care to wait for her colleague's response. At a first glance, mentioning a theorem can be related to the norm of mathematical efficiency, interpreted as: "all propositions should be justified".[2] The student, struggling between a 'practical' solution (since they had to draw few cases to justify their answer) and a 'theoretical' background that characterizes school geometry, comes up with one of the most familiar theorems. She has the need to contribute to the common task of writing an efficient solution and she offers the best she can at the given moment: Thales Theorem. By proposing something so novel at that moment she is exposed in front of her colleague, but mainly in front of the researcher, who was present. This is a typical face threatening act; and student B in order to minimize the potential damage to her positive face (in case her suggestion is proved to be inadequate) she immediately adds that "It doesn't fit". Student A, who seems to know the theorem, quickly rejects the plausibility of the theorem and they move on. Such exchanges are frequent in educational settings where many students feel that their positive face will be damaged in case of a wrong answer; and sometimes they choose to be silent and this has serious implications for their learning, but also for the establishment of the whole learning community.

Another remark on the given excerpt may come from the observation that student A seems to be in charge of the process; she expresses verbally the solution in [134], leaving to student B the role of merely writing it down, even when she thinks that their solution might not be complete [149]. The small excerpt contains some face empowering verbal acts, such as "what I say is correct" [140] and some face threatening acts for student B in [149] and [151]. Tatsis and Koleza (2006) while analyzing the interactions of three meetings of pairs of preservice teachers, observed the following roles:

(a) The dominant initiator: makes many suggestions, rarely asks for the partner's opinion and always tries to maintain face; demonstrates a low level of conformity to most social and sociomathematical norms, sometimes adjusts a norm by his/her acts. Whenever in a difficult position, attributes it to external factors (e.g. the difficulty of the task or even the inability of the partner).

(b) The collaborative initiator: makes many suggestions, asks for the partner's opinion, gives information whenever necessary and – most of the times – tries to maintain face; is ready to withdraw a suggestion but only if the opposing one is strongly grounded; generally demonstrates a high level of conformity to the social and sociomathematical norms established.

(c) The collaborative evaluator: makes relatively fewer suggestions compared to the previous roles, always gives information (whether asked or not) and tries to maintain face when s/he believes that it is not against any norm; thus, s/he shows a high level of conformity to the norms established and his/her acts demonstrate a high level of uniformity and facilitation to the partner's acts.

(d) The insecure conciliator: makes few suggestions and does not try to maintain face in an explicit way; shows a low level of conformity to most norms as s/he accepts his/her partner's suggestions without evaluating them; demonstrates the highest level of facilitation to the partner's acts. Whenever in a difficult position, attributes it to uncontrollable factors, such as ability or task difficulty.

The small presented excerpt does not provide sufficient amount of information to lead us to a valid characterization of the two students' roles. However, we may infer that student A is an initiator, and possibly a dominant one. Student B's behaviour, on the other hand, provides indications that she is a collaborative partner, but we cannot conclude whether she is an initiator or an evaluator.

Generally, in role theory, role performance is conceived as highly situated, thus the roles described before aimed to interpret the participants' act in the particular context (collaborative problem solving). What is important, however, is the fact that these roles are described in relation to the social and sociomathematical norms which are established. The importance of this fact lies in the realization that, while the role of the teacher is seen as central in the process of establishing the desired norms (Sfard, 2008), the actual process is much more complex due to the face needs of the participants (who fulfil these needs by performing various roles). I will return to this point in the final section of the paper.

The second issue that needs to be addressed is that a norm does not have to be accepted by all members of a community to be considered as such. Actually, as I already mentioned, the distribution and the conformity are just two attributes that can be used to describe a norm. This idea is not taken up by many contemporary researchers, who usually see a norm as a characteristic of the whole community of the classroom.[3] The first implication of this view is that other, 'minor' norms, which are enacted by few participants, are not given much attention. The second implication is that it creates an assumption that all norms which are desirable by the teacher are efficient for all students. Let me elaborate on this, by using as an example the norm of what counts as an insightful mathematical solution (Cobb & Yackel, 1996). It is quite apparent that the notion of 'insightful solution' is a rather subjective one, and even if the teacher establishes some metarules on that, there will still be some students for whom these rules will be inadequate.

CONCLUSIONS – AN ATTEMPT TO RECONCILE

In the preceding sections I have attempted to briefly present the basic characteristics of the concept of norm and then role theory with its relevant concept of face. I have also demonstrated how these concepts seem compatible, or even more, complementary, since they can be jointly applied in the analysis of interactions. A question might then arise: is such an approach justified by the theoretical assumptions of the relevant theories? The answer is positive, and it can be derived by examining some of the references in the papers mentioned before; they seem to have a common interest in the importance of interactions and many cite the theory of symbolic interactionism (Blumer, 1969). According to that theory, the role of symbols, especially language, is vital for the process of interactions; it is through symbols that people establish shared meanings and define the situation they are involved with. The person is not treated as a passive receiver of society's influences, but as an active participant who takes part in the formulation and negotiation of shared knowledge during the process of symbolic interaction.

The concept of norm is a fine tool for the researcher at the initial stage of the analysis, or for the teacher who wants to establish a feasible didactical contract with the students. However, once the researcher reaches the micro-level of utterances, s/he will probably notice that some of them do not seem to fit to any established norms; the researcher may also encounter some utterances that were not supposed to be heard in a mathematics classroom (e.g. because they are vague), which at the same time seem to be effective from a communicational point of view. The teacher might have more pragmatic worries concerning the enactment of the norms that should be established. How shall s/he deal with utterances like: "The maximum will probably be, er, the least 'll probably be 'bout fifteen." (Rowland, 2000, p. 1)? This utterance clearly violates the sociomathematical norm of clarity; however, this consideration cannot assist the teacher in establishing that norm, unless s/he becomes aware of the fact that the student might be using these linguistic forms in order to protect his face. Thus, acknowledging the face-saving needs of the participants might be of great importance in the mathematical classroom and particularly, in the establishment of social and sociomathematical norms.

NOTES

1. According to Sfard (2008) a rule is considered a norm only if it fulfils two conditions: it must be widely enacted within the discursive community and it must be endorsed by almost all members of that community, especially those considered as experts.

2. This can be also seen as an example of a negative influence of an established norm.

3. However, there is research (Planas & Gorgorió, 2004) indicating that in multi-ethnic mathematics classrooms students adhere to different sets of norms, which in turn results in learning obstacles for some of them.

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